

# Design optimization of system level adaptive optical performance

Gregory J. Michels\*, Victor L. Genberg, Keith B. Doyle, Gary R. Bisson  
Sigmadyne, Inc.  
803 West Avenue, Rochester, NY 14611

## ABSTRACT

By linking predictive methods from multiple engineering disciplines, engineers are able to compute more meaningful predictions of a product's performance. By coupling mechanical and optical predictive techniques mechanical design can be performed to optimize optical performance. This paper demonstrates how mechanical design optimization using system level optical performance can be used in the development of the design of a high precision adaptive optical telescope. While mechanical design parameters are treated as the design variables, the objective function is taken to be the adaptively corrected optical imaging performance of an orbiting two-mirror telescope.

**Keywords:** Finite Element, Optimization, Adaptive, Telescope

## 1. INTRODUCTION

Multidisciplinary optimization has been the focus of much recent work.<sup>1-6</sup> By linking predictive methods from multiple engineering disciplines, engineers are able to compute more meaningful predictions of a product's performance and improve the efficiency of the design process. In the design of precision optical systems, such as those used for imaging and communication, the optical performance is directly affected by the mechanical behavior of the system. Dynamic disturbances, thermoelastic deformation, and other operational loads cause the precisely formed optical surfaces of an optical system to move and deform. By coupling mechanical and optical predictive techniques mechanical design can be performed to optimize optical performance.

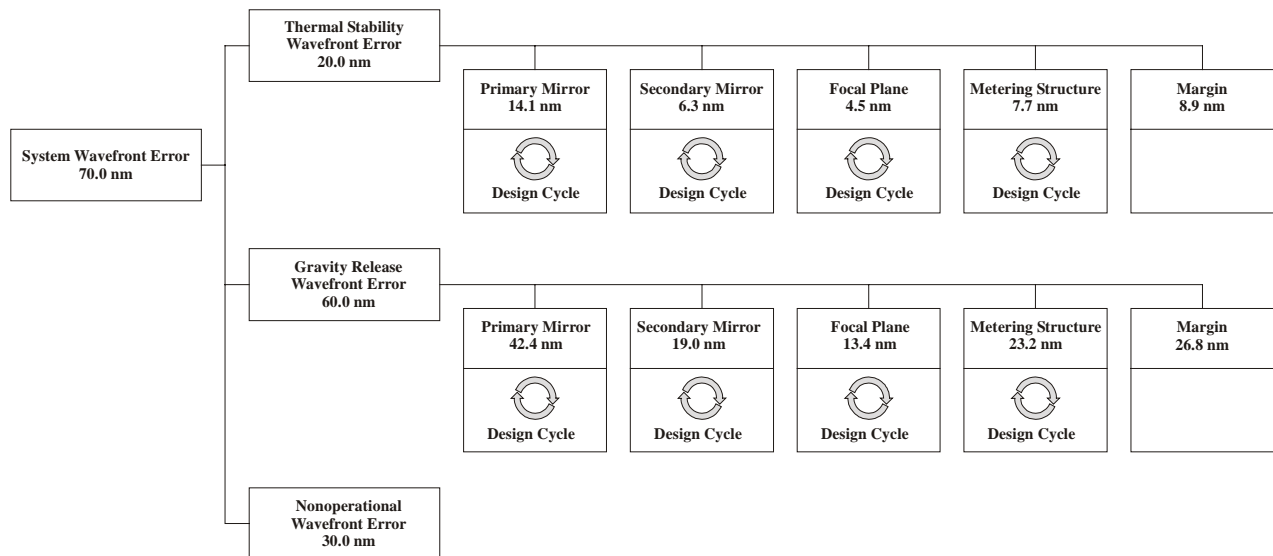


Figure 1: Example wavefront requirement budget for an optical system.

\* Correspondence: Email: michels@sigmadyne.com; www: <http://www.sigmadyne.com>; Telephone: (585) 235-6892

A common technique in the development of an optical system is to develop the mechanical design of individual subsystems by independent parallel processes. These designs are developed to requirements which have been flowed down from a top level system requirement as shown in Figure 1. This flow down of requirements to the various subsystems is done by experience and with much iteration as design teams negotiate for rebalancing of the allocations. This is done because prediction of an optical system's performance requires the use of optical design software with imported mechanical analysis results. Since such optical design tools are not integrated within any finite element tools in a widely available fashion for use in automatic design optimization, performing design trades on system level performance is cumbersome. While attempts have been made to integrate optical design software with mechanical finite element tools, there has been no widely available tool with which mechanical engineers can compute system level optical performance metrics within mechanical analysis software for a wide array of problems.<sup>2,4,6</sup>

This paper demonstrates how the DRESP3 feature can be used to predict the system wavefront error of a high precision adaptive optical imaging system to support design optimization in MSC.Nastran<sup>7</sup> SOL 200. While the weight of the primary mirror is minimized and mechanical design parameters are treated as the design variables, the adaptively corrected optical imaging performance of an orbiting telescope is held to a requirement.

## 2. ADAPTIVE OPTICS SIMULATION

It is of particular interest to be able to compute the corrected performance of an adaptive optical system. Since the ability of adaptive optical surfaces to deform into any desired shape is limited by their mechanical characteristics (e.g., stiffness and actuator layout), such predictive capability is required in designing the optical system to maximize the system's corrected performance. We introduce the methods of adaptive optics simulation in the following two subsections. The first deals with adaptive control of a single surface while the second deals with adaptive control of an entire optical system of multiple surfaces.

### 2.1 Single Surface Performance

Adaptive simulation of a single surface can be represented by the pictorial equation in Figure 2.<sup>8</sup> The first term on the left-hand side represents an input disturbance to the optical system which may be an aberrated incident wavefront or a deformation of the adaptive surface due to environmental loading. The other terms on the left hand side are called actuator influence functions and each represent the shape of the optical surface due to an arbitrary input of a single actuator. The term on the right hand side is the resulting surface due to the effect of the initial disturbance plus the effect of the actuators. The goal of adaptive simulation is to find the actuator control inputs  $x_1, x_2, \dots, x_n$  which minimizes the surface deformation on the right hand side.

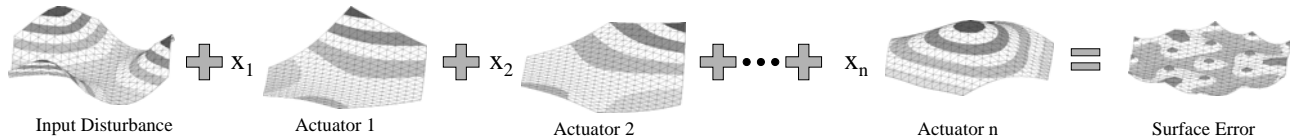


Figure 2: Pictorial equation of single surface adaptive simulation.

Mathematically, adaptive simulation of a single surface is achieved by minimizing the mean square surface error of the corrected optical surface. The corrected displacement of the  $i^{\text{th}}$  node,  $ds_i^{\text{Corr}}$ , is the sum of the uncorrected displacement of the  $i^{\text{th}}$  node,  $ds_i$ , and the displacements induced by each actuator,

$$ds_i^{\text{Corr}} = ds_i + \sum_j \mathbf{x}_j dx_{ji} \tag{2.1}$$

where  $x_j$  is the variable actuator input for the  $j^{\text{th}}$  actuator, and  $dx_{ji}$  is the displacement of the  $i^{\text{th}}$  node for the  $j^{\text{th}}$  actuator's influence function. The mean square residual error,  $E$ , of the corrected optical surface is represented by,

$$E = \sum_i w_i \left( ds_i + \sum_j x_j dx_{ji} \right)^2, \quad (2.2)$$

where  $w_i$  is the area weighting of the  $i^{\text{th}}$  node. Area weighting is discussed in more detail by Genberg.

The actuator inputs that minimize the mean square error,  $E$ , are found by taking derivatives of Eq. (2.2) with respect to each actuator input,  $x_j$ , and setting each resulting equation equal to zero. This results in the following linear system represented by Equ. (2.3):

$$[H]\{X\} = \{F\}, \quad (2.3)$$

where

$$H_{jk} = \sum_i w_i dx_{ji} dx_{ki}, \quad (2.4)$$

and

$$F_k = \sum_i w_i ds_i dx_{ki}. \quad (2.5)$$

The adaptive mirror's residual RMS surface error,  $\sqrt{E}$ , may be computed with the use of Equ. (2.2) once the actuator inputs have been found from Equ. (2.3), (2.4), and (2.5).

## 2.2 System Level Performance

While it may be useful in some applications to predict the adaptive control performance of a single optical surface, a much more powerful tool is to be able to predict the adaptive performance of an entire optical system as discussed in Section 1. The approach used here is based upon the method of superposition and the linearity of Zernike polynomials in the wavefront with Zernike polynomials and rigid body motions describing the deformations and motions of optical surfaces.<sup>9</sup> For the small surface perturbations typical of adaptive optical systems, this approximation is generally valid. This computation is performed by importing the changes in the wavefront error of each Zernike polynomial at a location of optical performance measurement (e.g., the focal plane) with respect to changes in the rigid body motion and Zernike polynomials of surface deformation of each optical surface in the system. These sensitivities can be computed in an automated fashion by optical design software. With this matrix of information we are able to compute the performance of the optical system due to any deformations and motions of the optical surfaces as long as they can be described by the rigid body motions and Zernike polynomials used in the development of the optical sensitivity matrix. Notice that the optical sensitivity matrix needs only to be generated once before any mechanical analysis as long as the optical prescription (shape and location of the optical surfaces) does not change. This method of system level optical performance prediction can be extended to adaptive control prediction in a similar method as was used in Section 2.2.

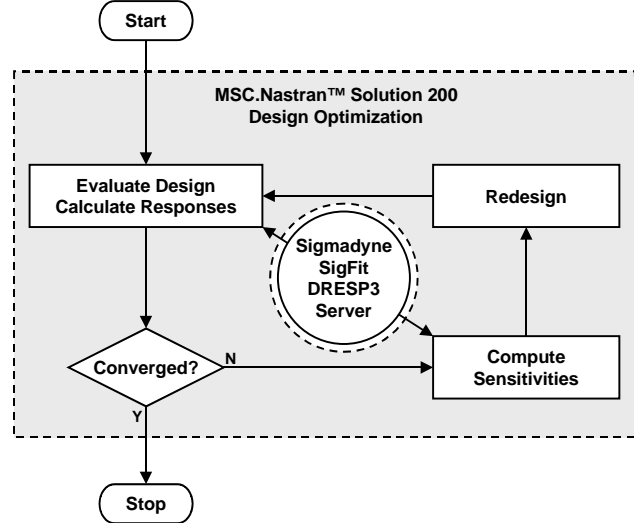


Figure 3: Flowchart of design optimization with system level optical performance evaluation.

The mathematical description for performing integrated adaptive analysis using Zernike decomposition techniques is provided below. An optical sensitivity matrix,  $S_{kj}^n$ , represented by a set of Zernike terms,  $k$ , at the location of optical performance measurement, is computed from the optical system analysis model by perturbing each optical surface,  $n$ , by a unit surface perturbation,  $j$ . Each of the unit surface perturbations is either a rigid body motion or a Zernike polynomial. Enough of these perturbations are chosen to adequately describe the expected deformations of the optical surfaces.

Once the optical sensitivity matrix has been generated, each actuator influence function,  $\Phi_m$ , where  $m$  is the actuator, is decomposed into rigid body motions and Zernike polynomials,  $B_{jm}$ , by simple polynomial fitting, where,  $j$  is the rigid body motion or Zernike polynomial term fit to the influence function. Notice that both  $\Phi_m$  and  $B_{jm}$  may contain the influence functions from multiple optical surfaces if there is more than one adaptive surface in the system. This polynomial fit is then converted to actuator influence functions of optical performance by,

$$U_{km} = S_{kj}^n B_{jm} . \quad (2.6)$$

For a given perturbed state of the optical system due to an environmental load, each deformed surface is similarly decomposed into Zernike coefficients,  $C_{ji}^n$ , where,  $j$ , is the vector of Zernike coefficients for load case,  $i$ , of each optical surface,  $n$ . The wavefront error of the optical system is computed by multiplying the optical sensitivities by the Zernike terms of the deformed surface

$$Z_{ki} = S_{kj}^n C_{ji}^n \quad (2.7)$$

where  $Z_{ki}$  is vector of Zernike polynomials,  $k$ , for each load case,  $i$ .

Similar to Equa. (2.2) fitting the actuators to minimize the optical system wavefront error is performed using a least-squares fit. A system error,  $E$ , is defined as

$$E = \sum_k^Z w_k \left( Z_{ki} - \sum_m^M U_{km} A_m \right)^2 , \quad (2.8)$$

where  $A_m$  is the contribution coefficients of each actuator and  $w_k$ , is the wavefront RMS error of each polynomial term,  $k$ , at the optical performance measurement location.

To compute the best-fit actuator coefficients, the error function is minimized with respect to the actuator contribution coefficients. This is performed by taking the derivative of the error function with respect to these coefficients and setting it equal to zero as follows:

$$\frac{dE}{dA_q} = \sum_k^Z w_k 2 \left( Z_{ki} - \sum_m^M U_{km} A_m \right) U_{kq} = 0 . \quad (2.9)$$

Rearranging terms yields the following expression:

$$\sum_k^Z w_k \left( \sum_m^M U_{km} U_{kq} A_m \right) = - \sum_k^Z w_k (Z_{ki}) U_{kq} . \quad (2.10)$$

The actuator coefficients may be solved via the following matrix equation:

$$[H]\{A\} = \{F\} , \quad (2.11)$$

where

$$H_{qm} = \sum_k^Z w_k U_{qk} U_{km}, \quad (2.12)$$

and

$$F_q = -\sum_k^Z w_k Z_k U_{kq}. \quad (2.13)$$

The adaptive mirror's residual RMS surface error,  $\sqrt{E}$ , may be computed with the use of Equ. (2.8) once the actuator inputs have been found from Equ. (2.11), (2.12), and (2.13).

### 3. IMPLEMENTATION

The DRESP3 subroutine used to implement the methods in Section 2 was used to return the corrected surface RMS error,  $\sqrt{E}$ . The subroutine is a modified form of Sigmadyne's commercially available software, *SigFit*. An additional utility was written to write the DRESP3 and other required bulk data.

#### 3.1 Bulk Data Format

A combination of a DTABLE card and DRESP1 cards were used with the DRESP3 feature.<sup>10</sup> The DTABLE card is used to store the various user selectable option values, prescription data associated with each optical surface, optical surface node locations, and optical surface node area weighting. If the user chooses to perform system level analysis instead of single surface analysis, then the optical surface sensitivity matrix, **S**, is stored on the DTABLE as well. An edited example DTABLE card is shown in Figure 4.

DTABLE	ina	1.0	nsym	0.0	nsbc	0.0	ns	3.0
	ngs01	204.0	roc01	-1.015+2	ckk01	-1.00230	ssc01	1.0
	ngs02	145.0	roc02	-1.249+1	ckk02	-1.49690	ssc02	1.0
	ngs03	1.0	roc03	0.0	ckk03	0.0	ssc03	1.0
	irb	0.0	izd	0.0	wvl	3.9370-8	izk	5.0
	nat	9.0	iaa	0.0	ndt	1.0	isys	1.0
	x00001	2.62500	y00001	-1.51554	z00001	-4.520-2	w00001	4.9020-3
	x00002	1.50000	y00002	0.0	z00002	-1.110-2	w00002	4.9020-3
	.							
	x00349	1.44938	y00349	-0.38836	z00349	-9.046-2	w00349	6.8966-3
	x00350	0.0	y00350	0.0	z00350	0.0	w00350	1.00000
	ism01	3.0	nzp01	37.0				
	a00001	0.0	a00002	0.0	a00003	0.0	a00004	0.0
	a00005	0.0	a00006	0.0	a00007	-1.135+4	a00008	-1.84-13
	.							
	a00217	3.3925-6	a00218	2.4075-6	a00219	-5.857-6	a00220	-2.017-3
	a00221	-2.016-3	a00222	0.0				
	b00001	0.0	b00002	0.0	b00003	0.0	b00004	0.0
	b00005	0.0	b00006	0.0	b00007	0.0	b00008	0.0
	.							
	b01365	2.1019-4	b01366	1.1025-3	b01367	1.1025-3	b01368	-5.224-3
	b01369	1.98370						
	ism02	3.0	nzp02	37.0				
	c00001	0.0	c00002	0.0	c00003	0.0	c00004	0.0
	c00005	0.0	c00006	0.0	c00007	1.0250+4	c00008	-3.58-14
	.							
	.							

Figure 4: Example DTABLE card for system level adaptive control analysis.

DRESP1 cards are used to compute the optical surface nodal translations. Nodal rotations are not required. A set of DRESP1 cards covering all optical surface nodes are written for each actuator influence function subcase and disturbance subcase. Example DRESP1 cards are shown in Figure 5.

DRESP1	100001D0100001	DISP	1	10001
DRESP1	100002D0100002	DISP	2	10001
DRESP1	100003D0100003	DISP	3	10001
DRESP1	100004D0100004	DISP	1	10003
DRESP1	100005D0100005	DISP	2	10003
DRESP1	100006D0100006	DISP	3	10003
DRESP1	100007D0100007	DISP	1	10006
DRESP1	100008D0100008	DISP	2	10006
DRESP1	100009D0100009	DISP	3	10006
.				
.				

Figure 5: Example DRESP1 cards for request of translational displacements of optical surface nodes.

The DRESP3 card is simply written to reference the DTABLE and DRESP1 entries so that they may be exported to the DRESP3 server routine. All of these cards are written by the utility program mentioned above with input from the user.

The DRSPAN card must then be used in case control to assign the sets of DRESP1 cards to their respective subcases as shown in Figure 6.

SUBCASE 11
LABEL=DISPLACEMENT ACTUATOR #1
LOAD=24101
SET 11 = 100001 THRU 101050
DRSPAN=11
SUBCASE 12
LABEL=DISPLACEMENT ACTUATOR #2
LOAD=24201
SET 12 = 200001 THRU 201050
DRSPAN=12
SUBCASE 13
LABEL=DISPLACEMENT ACTUATOR #3
LOAD=24301
SET 13 = 300001 THRU 301050
DRSPAN=13
.
.

Figure 6: Example subcases showing the use of the DRSPAN card.

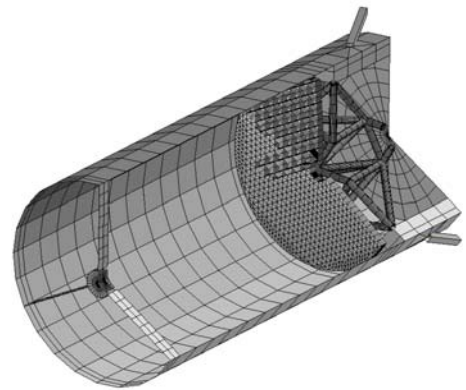


Figure 7: Finite element model of orbiting telescope.

### 3.2 Example

An example telescope model is used to demonstrate the optimization of an orbiting telescope's adaptive primary mirror. A plot of the telescope model is shown in Figure 7. The primary mirror is of lightweighted construction composed of a lightweighted triangular cell core with front and back faceplates. A primary mirror reaction structure provides mounting locations for three displacement actuators and six force actuators. The layout of the mounts and actuators on the mirror is shown in Figure 8. The primary mirror reaction structure also supports the focal plane unit behind the primary mirror. A cylindrical shell is used to meter a secondary mirror assembly comprised of the secondary mirror and spider structure. Six main struts are used to mount the telescope to the spacecraft bus.

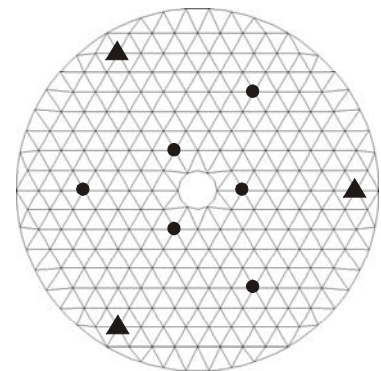


Figure 8: Actuator layout. Triangles show the displacement actuators while the circles show the push-pull force actuators.

The goal of the optimization is to minimize the weight of the adaptively controlled primary mirror while constraining the optical performance of the telescope as shown in Figure 1. The optical performance is measured by the wavefront error at the exit pupil of the telescope system. Several design variables relating to the design of the primary mirror are defined. These variables

include the depth of the mirror core, the thicknesses of the facesheets, and the wall thicknesses of various regions of the mirror core as shown by the shading in Figure 9. The thickness variables were defined such that thicknesses could be designed near each of the mounts and actuators. Additional requirements were imposed consisting of a minimum natural frequency and a launch stress allowable. The optimization problem is formally defined as follows:

**MINIMIZE:**

Weight of primary mirror

**DESIGN VARIABLES:**

- Optical facesheet thickness:  $0.18 \text{ inch} < t_f < 0.25 \text{ inch}$
- Back facesheet thickness:  $0.10 \text{ inch} < t_b < 0.25 \text{ inch}$
- Interior core wall thicknesses:  $0.04 \text{ inch} < t_c < 0.25 \text{ inch}$
- Inner and outer core wall thicknesses:  $0.08 \text{ inch} < t_c < 0.25 \text{ inch}$
- Core depth:  $0.25 \text{ inch} < t_c < 5.0 \text{ inch}$

**SUBJECT TO:**

- Thermally induced system wavefront error  $< 20 \text{ nm RMS}$
- Gravity release induced system wavefront error  $< 60 \text{ nm RMS}$
- Peak launch induced stress in PM  $< 1000 \text{ psi}$
- First mounted PM natural frequency  $> 200 \text{ Hz}$

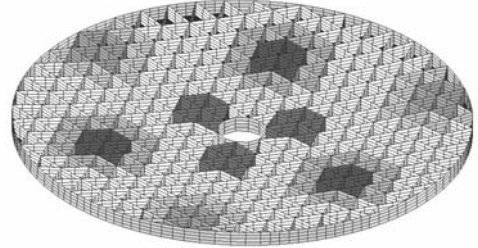


Figure 9: Core thickness variables shown by shading.

The analysis results for the initial design and the optimized design are shown in Table 1 alongside the requirements. Notice that the optimizer reduces the weight of the primary mirror by over 50% while the constraints on system wavefront error, launch stresses, and natural frequency are obeyed. It is important to notice that the stress constraint is already active in the initial design while the gravity induced wavefront error constraint is nearly active.

Table 1: Results of Design Optimization

Response	Initial Design	Optimized Design	Requirement
Thermally Induced Wavefront Error	9 nm	20 nm	20 nm
Gravity Release Induced Wavefront Error	54 nm	60 nm	60 nm
Peak Launch Stresses	1000 psi	1000 psi	1000 psi
First Natural Frequency	231 Hz	221 Hz	200 Hz
Weight	20.8 kg	9.9 kg	Minimum
Areal Density	53.0 kg/m <sup>2</sup>	25.2 kg/m <sup>2</sup>	Minimum

Notice that the use of the system level optical performance as a response allows the design of the primary mirror to develop into a design which best corrects the system wavefront error rather than only the wavefront error of the primary mirror assembly. The optimum primary mirror design, therefore, includes the best mix of design characteristics for correcting the errors from the secondary mirror assembly, the errors from the metering structure, as well as the errors from the primary mirror assembly. This approach gives the optimizer more freedom in finding an optimum design compared to the method of Figure 1 which optimizes each subsystem independently to separately allotted requirements.

#### 4. CONCLUSIONS

A method for incorporating multidisciplinary optimization in the design process of optical systems was presented. The process is highlighted by being able to predict the system level optical performance during automated optimization of the mechanical design. The use of this system level performance in design optimization allows the optimizer to find designs which might be unrealizable by independent subsystem optimization. This process would be most effective early in the design process when various system concepts are being compared. This method allows engineer's to quickly obtain a rough estimate of the optimum design for the system. Once this has been done for each concept being considered a final selection of the best concept can be made. The resulting subsystem designs and error allocations found by the optimizer can then be given to separate design teams for more detailed design work to be performed in parallel.

## 5. REFERENCES

1. Doyle, Genberg, Michels, **Integrated Optomechanical Analysis**, SPIE Press, Bellingham, WA, 2002.
2. B. Cullimore, T. Panczak, J. Baumann, V. Genberg, M.Kahan, "Integrated Analysis of Thermal/Structural/Optical Systems", ICES Paper, 2002.
3. Gregory J. Michels, Victor Genberg, "Design Optimization of Actively Controlled Optics", SPIE Paper 4198-17, November 2000.
4. A. Williams, V. Genberg, S. Gracewski, B. Stone, "Simultaneous Design Optimization of Optomechanical Systems", SPIE Paper 3786-28, 1999.
5. V. Genberg, "Optical Performance Criteria in Optimum Structural Design", SPIE Paper 3786-29, July 1999.
6. A. Williams, V. Genberg, S. Gracewski, B. Stone, "Multidisciplinary Design Optimization of Optomechanical Devices", Proc 3<sup>rd</sup> WCSMO, 1999.
7. NASTRAN is a registered trademark of the National Aeronautics and Space Administration. MSC.Nastran is a registered trademark of the MSC.Software corporation.
8. V. Genberg, G. Michels, "Opto-mechanical analysis of segmented/adaptive optics", SPIE Vol. 4444 (10), San Diego, August, 2001
9. K. Doyle, V. Genberg, G. Michels, "Integrated optomechanical analysis of adaptive optical systems", SPIE Vol. 5178 (5), San Diego, CA, August, 2003.
10. Johnson, Erwin H. et al., **MSC.Nastran 2004 Design Sensitivity and Optimization User's Guide**, MSC.Software, Santa Ana, CA, 2003.