# Optical modeling of finite element surface displacements using commercial software

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## ABSTRACT

The accuracy of optical modeling techniques to represent finite element derived surface displacements is evaluated using commercial software tools. Optical modeling methods compared include the Zernike polynomial surface definition, surface interferogram files, and uniform arrays of data in representing optical surface errors. Methods to create surface normal displacements and sag displacements from FEA displacement data are compared. Optical performance evaluations are performed as a function of surface curvature (*f*/#). Advantages and disadvantages of each approach are discussed.

Keywords: opto-mechanical analysis, integrated modeling, optical surface displacements, interferogram files, finite element analysis

## 1. INTRODUCTION

Predicting optical performance over the operational environment of an optical system often requires importing finite element computed surface displacements into the optical model. This may include predicting surface deformations in service environments due to inertial and thermal loads to on-orbit random vibrations to predicting performance of an adaptive optical system. In general, for each of the above mechanical analyses, the modeling process first requires computing the mechanical displacements of the optical surface using finite element analysis, second post-processing the FEA displacement data into the appropriate optical displacement value, and third, representing the surface errors in the optical model using various optical modeling methods.

Common optical modeling methods used to represent surface errors in commercially available optical design software such as CODEV[1] and ZEMAX[2] include polynomial surface definitions, surface interferogram files, and uniform arrays of data. General discussion and application of representing finite element surface displacements using the above optical modeling techniques is by discussed by Doyle et al[3]. These methods are reintroduced in Section 2 and their ability to accurately represent surface errors is compared in Sections 4 and 5. These optical modeling methods use either sag or surface normal displacements. (The sag displacement is defined as the distance from the vertex tangent plane to the optical surface). Sag and surface normal displacement vectors are shown in Figure 1.1.



Figure 1.1 Arrows define the sag (left) and surface normal displacement (right) direction.

Linear and nonlinear methods to compute sag displacements from the FEA displacements are discussed in Section 3 and performance compared in Sections 4 and 5.

Sag and surface normal displacements comprise only a subset of the full FEA computed displacement vector. Neither set of displacements accurately represent decenter or in-plane rotation of an optical surface. Thus the rigid-body decenter and in-plane rotation must be removed from the FEA surface displacements prior to computing the sag and surface normal displacements. The rigid-body errors of the optical surface may then be represented as tilts and decenters in the optical model. Conversely, all six rigid-body surface errors may be removed from the set of surface displacements and represented using tilts and decenters. The elastic surface displacements may be represented using the various surface modeling techniques discussed here. This approach allows optical performance degradation to be evaluated due to each of the six rigid-body perturbations and elastic surface errors individually. A comparison of these two modeling approaches in representing rigid-body errors is presented in Section 4.

The optomechanical analysis software package SigFit[4] provided a convenient tool to make the comparison studies. SigFit options include converting FEA displacement data into linear sag displacements, nonlinear sag displacements, or surface normal displacements; fitting an arbitrary order of Zernike polynomials; removing rigid-body terms prior to polynomial fitting; creating uniform arrays of data using linear or cubic interpolation routines; and creating output files in both CODEV and ZEMAX formats.

## 2. OPTICAL MODELING OF SURFACE DISPLACEMENTS

There are several common methods using commercial optical design software to represent finite element derived optical surface displacements. These methods are discussed below.

### 2.1 Polynomial Surface Definition

Polynomial surface definitions allow finite element displacement data to be fit to polynomials and added as perturbations to a base surface definition. Polynomial options typically include Zernike polynomials, x-y polynomials, polynomial aspheres, and others. Changes to the optical surface definition are defined by changes in the sag value. This requires the finite element computed surface displacements to be converted into sag displacements. For example, the finite element derived sag deformations may be represented by Zernike coefficients as perturbations to the base surface shown below:

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2 r^2}} + \sum a_i Z_i \qquad (2.1)$$

where z is the sag of the optical surface, the first term is the nominal surface definition, and the second term are the perturbations to the base surface represented by the Zernike coefficients, $a_i$, and the Zernike polynomials, $Z_i$.

The limitation in this approach is the accuracy of the polynomial fit to the sag displacements. For example, the number of Zernike terms used in CODEV's SPS ZRN (10th-order Standard Zernike polynomial) and ZEMAX's Zernike Standard Sag surface (20th-order Standard Zernike polynomial) is 66 and 231, respectively. The surface representation is an approximation if the 66 or 231 Zernike terms do not exactly represent the sag displacements.

## 2.2 Surface Interferogram Files

Surface interferogram files are CODEV native and are two-dimensional data sets that are assigned as deviations to an optical surface. The data is assumed normal to the optical surface requiring finite element displacements to be converted into surface normal displacements. Interferogram file data may be represented in two formats - Zernike polynomials (Standard or Fringe) or as a uniform rectangular array (discussed below). CODEV places no limit on the number of Zernike polynomial terms that may be used to represent the surface normal displacements. The interferogram file data may be scaled in the optical model using a scale factor command which is useful in performing design trades by scaling surface errors due to unit g-loads, thermal soaks, or gradients.

This format is an approximate technique to represent a deformed surface shape. The approximation lies in the computation of the optical errors for a given ray. A ray is traced to the undeformed surface, and the intersection coordinates are used to determine the surface error as defined by the interferogram file from which ray deviations and OPD are computed. The error associated with this approximation is a function of the ray angle and the spatial variation and magnitude of the displacement field. The error in this approximation in representing FEA optical surface deformations is typically small.

## 2.3 Uniform Arrays of Data

Uniform arrays of data are useful in representing optical surface displacements when an accurate polynomial fit cannot be achieved. Arrays are able to represent high frequency spatial variations seen in edge roll-off, localized mounting effects, or quilting of a lightweight optic. For example, the surface displacements shown in Figure 2.1 are the resulting surface error of a lightweight optic due to gravity and a uniform temperature change after adaptive correction. The percent of the rms surface error represented by a 66-term and 231-term Standard Zernike polynomial is shown in Table 2.1. A large fraction of the surface displacements is not included in the Zernike fit for each of these two cases. A uniform array provides a much more accurate representation. For example, a 51 x 51 array represents over 98% and 99% of the rms surface error for the two cases, respectively.
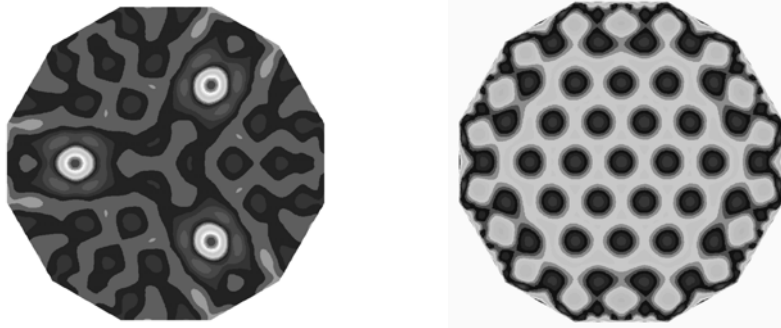


Figure 2.1 Surface displacements due to gravity (left) and thermal soak (right) after adaptive correction.

|  | 66-Term Fit | 231-Term Fit | Grid 51x51 |
|---|---|---|---|
| Gravity | 5% | 32% | 98% |
| Thermal Soak | 4% | 40% | 99% |

Table 2.1 Percent of rms surface error represented by 66 and 231-term Standard Zernike polynomial and a 51 x 51 uniform array.

The loss in accuracy in representing surface displacements with uniform arrays of data is twofold; first, in general, interpolation is required to create a uniform rectangular array from a non-uniform FEA mesh; and second, errors result from ray tracing in the optical model for incident rays that do not coincide with a data point. In this case, a second interpolation step is used within the optical model to define the surface errors. Two common uniform array formats are CODEV's surface interferogram files (see above) and the Zemax Grid Sag surface discussed below.

### 2.3.1 Grid Sag Surface

The Grid Sag Surface is a Zemax surface definition that uses a uniform array of sag displacements and/or slope data to define perturbations to a base surface as defined below

$$z = z_{base} + z(x_i, y_j).$$ (2.2)

ZEMAX offers two interpolation routines, linear and bicubic, to determine the surface errors during optical ray tracing. If only sag displacements are provided, the user may select the linear interpolation routine or use bicubic interpolation where ZEMAX computes the slope terms using finite differences. If in addition to the sag displacements, the first derivatives in the x and y-directions, and the cross-derivative terms are supplied by the user, ZEMAX's bicubic interpolation may be used.

## 3. COMPUTING SAG AND SURFACE NORMAL DISPLACEMENTS

The optical modeling methods presented in the previous section requires the finite element surface displacements to be converted into either sag or surface normal displacements. The manner in which the FEA data is converted represents a source of error. Approximate techniques using small displacement theory to compute the sag and surface normal displacements are discussed in detail by Genberg and Michels[5]. (Note that the sag displacement does not equal the FEA computed Z-displacement if the node is also displaced in the radial direction e.g. thermal loads.) These values represent the deviation of the deformed optical surface from the undeformed optical surface at each initial node position and are denoted as $\Delta Sag_1$ and $\Delta$Surface Normal in Figure 3.1. These sag and surface normal displacements may be scaled which provides efficiencies for trade studies and multiple load combinations created from unit g-loads, thermal soaks, and thermal gradients applied to the FEA model. The linearization is also advantageous for active control simulation and predicting surface errors due to dynamic loading using modal techniques. Without a linear relationship these analyses would not be possible.

A method to compute an exact sag displacement value is presented by Juergens and Coronado[6,7]. This value represents the deviation of the deformed optical surface to the undeformed optical surface from the displaced node position and is depicted as $\Delta Sag_2$ in Figure 3.1. These sag values are not a linear function of the deformed surface and thus may not be scaled. For this reason they are referred to in this paper as nonlinear sag displacement values to differentiate from the linear computed sag values discussed above. Differences between the nonlinear and linear sag computations arise only when surface nodes are radially displaced such as surfaces under thermal loading. If there is no radial displacement, both the linear and nonlinear sag displacements equal the z-displacement.
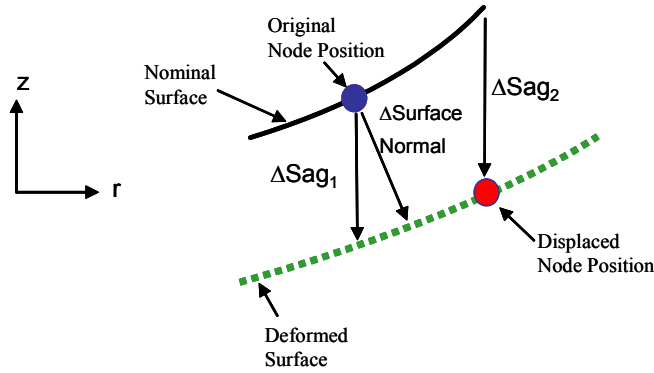
Figure 3.1 Linear and nonlinear sag displacements and surface normal displacements.

The current method used to compute the surface normal displacements can only approximate the FEA predicted shape change of an optical surface. Consider a surface undergoing a pure unit shift in the z-direction, $\Delta Z$ (despace), as depicted in Figure 3.2. For a node at the surface vertex, the surface normal displacement ($\Delta SN$) equals the z-direction rigid-body displacement. For all other points on the surface, the surface normal is computed as the cosine of the local angle of the surface normal vector with the local z-axis (dot product of the FEA displacement vector with the surface normal unit vector). For nodes further away from the optical axis, the angle increases, and the surface normal displacement decreases. This method used to calculate the surface normal excludes the displacement component tangent ($\Delta ST$) to the optical surface. As surface curvature increases, the error in using surface normal displacements to represent FEA derived surface errors increases.
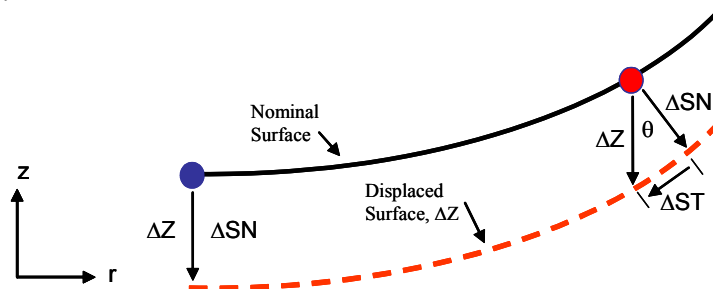


Figure 3.2 Computing the surface normal displacement for vertex and non-vertex points.

## 4. CASE STUDIES

Optical modeling techniques and FEA conversion methods were compared on a 12-inch diameter parabolic mirror of various surface curvatures ($f/\#$). Finite element models were analyzed using MSC/NASTRAN and optical analysis was performed using ZEMAX and CODEV. The FEA surface displacements were converted into sag and surface normal displacements, fit to Zernike polynomials and uniform arrays, and output in the specific format required by CODEV and ZEMAX using SigFit. Case studies evaluated included; 1) rigid-body errors; 2) Zernike elastic surface errors; and 3) thermal soaks.

The geometric rms spot size diameter was used to compare the ability of these optical modeling methods to represent finite element surface displacements. These analyses assumed an object at infinity and a wavelength of 0.5 μm. All rms spot size results are provided in inches. The rms spot size predicted by the various modeling methods is compared to exact results based on perturbing the optical surface directly in the

optical software package. For example, for rigid-body errors, the optical surface is tilted or despaced directly in the optical model; for known Zernike imposed surface errors, exact results are determined using the Zernike polynomial surface definition; and for thermal soaks, the FEA deformed surface representation is compared to a surface whose radius of curvature has been modified to the known value. For all test cases, the nodes of the finite element model were defined to be on the optical surface using double precision.

## 4.1 Rigid-Body Errors

The rigid-body errors decenter, despace, and tilt were applied to the FEA model of an $f/1$ parabola as illustrated in Figure 4.1. Two methods to represent rigid-body errors were compared. The first approach includes the rigid-body errors in the surface representation. In the second method, the rigid-body terms were removed from the surface representation and included as tilts and decenters in the optical model.

In the first approach, the Zernike polynomial surface using nonlinear sag displacements and the Zernike surface interferogram file (surface normal displacements) were used to represent the rigid-body errors despace and tilt as applied to the FEA model. (Decenter results cannot be represented accurately and are always removed). Use of the linear sag displacements are not considered since the computations in SigFit using linear sag displacements assume all rigid-body errors are removed and represented as tilts and decenters in the optical model.



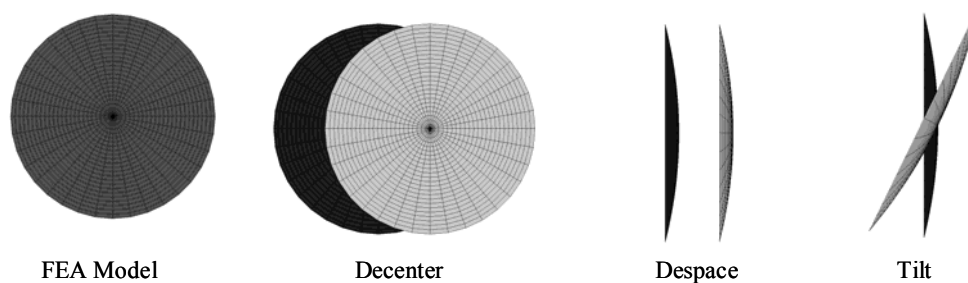|     FEA Model     |     Decenter     |     Despace     |     Tilt     |

Figure 4.1 FEA model and rigid-body decenter, despace, and tilt errors.

The rms spot size results and percent error relative to the exact value are shown in Table 4.1. The exact rms spot size is computed by directly applying the FEA imposed rigid-body error directly on the optical surface using tilts and decenter commands.

| Rigid-Body Motion | Exact | Sag Nonlinear / % Error | Surface Normal / % Error |
|---|---|---|---|
| Despace 0.01" | 7.3826E-03 | 0.0073826 / 0.00 | 0.0011233 / 52.2 |
| Tilt 1.0-Deg | 2.3968E-02 | 0.023971 / 0.01 | 0.026238 / 9.5 |

Table 4.1 Comparison of rms spot sizes and percent error with rigid-body errors included in the set of surface displacements.

The representation of the rigid-body errors using surface interferogram files is more than 50% off the exact value for despace and almost 10% off for tilt. The error in using the nonlinear sag displacements is 0.01% in representing the rigid-body tilt.

In the second approach, the rigid-body terms from the set of FEA surface displacements were removed and represented as rigid-body surface errors in the optical model using tilts and decenters.  Any residual surface errors are represented using the Zernike polynomial surface definition with nonlinear and linear sag displacements, and Zernike surface interferogram files.  The rms spot size results for the three modeling approaches are shown in Table 4.2.

| Rigid-Body Motion | Exact | Sag Nonlinear | Sag Linear | Surface Normal |
|---|---|---|---|---|
| Decenter 0.01" | 1.3452E-15 | 2.0262E-15 | 3.2882E-11 | 3.3567E-11 |
| Despace 0.01" | 7.3826E-03 | 7.3826E-03 | 7.3826E-03 | 7.3826E-03 |
| Tilt 1.0-Deg | 2.3968E-02 | 2.3968E-02 | 2.3968E-02 | 2.3968E-02 |

Table 4.2 Comparison of rms spot sizes with rigid-body errors removed from the set of surface displacements.

For both the despace and tilt case results, all three methods yield the exact result out to the number of digits in the calculation.  A comparison of the decenter results show that each of the methods yields essentially an infinitesimal spot size with the nonlinear sag displacements values more accurate.  In general when representing rigid-body surface errors, the most accurate modeling technique is to remove the rigid-body errors from the set of FEA surface displacements and use tilt and decenter commands to represent the errors in the optical model.  The only accurate modeling technique where rigid-body terms may remain is the use of the nonlinear sag displacements in the polynomial surface definition.

## 4.2 Elastic Surface Errors

Elastic surface errors were applied to the FEA model as surface sag deformations in the form of Zernike polynomial terms.  Applied Zernike surface errors including focus, astigmatism, coma, trefoil, and spherical aberration as shown in Figure 4.2.  For each case, the applied Zernike coefficient is 0.0625 waves yielding a 0.125 p-v surface error.  CODEV surface interferogram files (Zernike and grid format) and the ZEMAX Grid Sag surface were used to represent the surface errors in the optical model.  The exact rms spot size to which the results are compared was provided by using CODEV's and ZEMAX's Zernike polynomial definition.



Figure 4.2 Applied Zernike Surface Displacements from left to right; focus, astigmatism, coma, trefoil, and spherical aberration.

4.2.1 Elastic Surface Errors: Surface Interferogram Files

CODEV surface interferogram files (Zernike and grid format) were used to represent the elastic surface errors.  This study was performed as a function of $f/\#$ and for the case of the grid interferogram files, a function of grid density.

For each of the five load cases, surface normal displacements were computed at each node point in the FEA model. Zernike Surface interferogram files were created by using Standard Zernike polynomials with 66-terms (fit error was <0.01%). Uniform arrays of data for the grid surface interferogram file were created of various grid densities using SigFit's cubic interpolation routine.

A comparison of the percent error of the Zernike surface interferogram files representing each of the Zernike applied surface deformations is shown in Figure 4.3. For highly curved surfaces, the ability of the surface normal to represent the Zernike surface errors decreases. For example, the maximum percent error exceeds 20% for spherical aberration with the $f/0.5$ surface. The percent error drops significantly as the surface curvature decreases. For the $f/1.0$ surface, the maximum percent error is approximately 5%, and for the $f/1.5$ surface the maximum percent error drops to approximately 2%.

Surface interferogram files using a rectangular array of data (grid format) were evaluated in representing the Zernike surface errors as a function of grid density for the $f/1.0$ surface. The grid interferogram file is evaluated in terms of a percent error relative to the Zernike format since as the grid density increases the surface approximates the continuous surface defined by the Zernike polynomials. SigFit's cubic interpolation routine was used to create the grid arrays. The results are shown in Figure 4.4. Using a 401 x 401 uniform grid, the maximum percent error for the five surface errors is approximately 2.6% in representing spherical aberration. The other four Zernike surface errors show less than 1% error. Slight improvement is seen in increasing the grid array to 801 x 801.
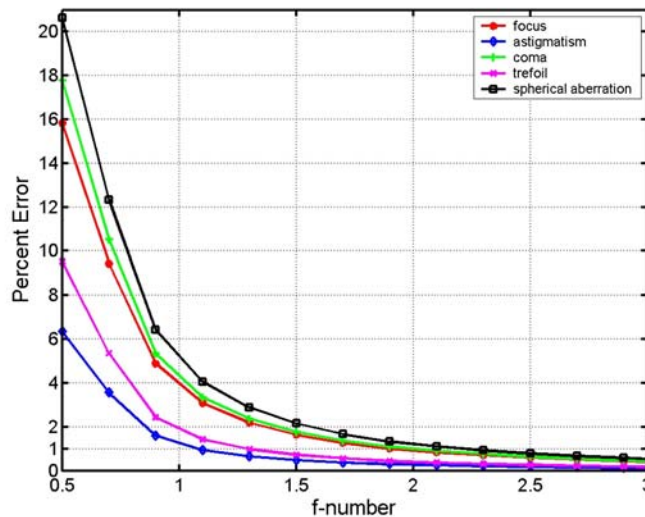


Figure 4.3 Zernike surface interferogram files percent error as a function of $f$-number.

4.2.2 Elastic Surface Errors: Grid Sag Surface

The Zemax Grid Sag surface was used to represent Zernike surface errors imposed on the $f/1.0$ parabola as a function of grid density. The Grid Sag surface uses sag displacements and thus the accuracy of the representation is not a function of $f$-number as is the case of surface normal displacements. The percent error of the grid sag surface relative to the exact value as a function of grid density is shown in Figure 4.5. SigFit's cubic interpolation routine was used to create the grid arrays. Using a 401 x 401 uniform grid, the maximum percent error for spherical aberration is ~1.7%. The other four Zernike surface errors have less than 1% error.
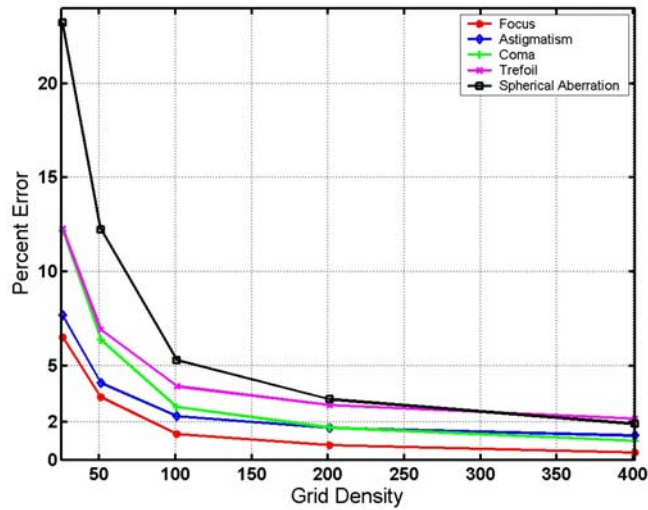
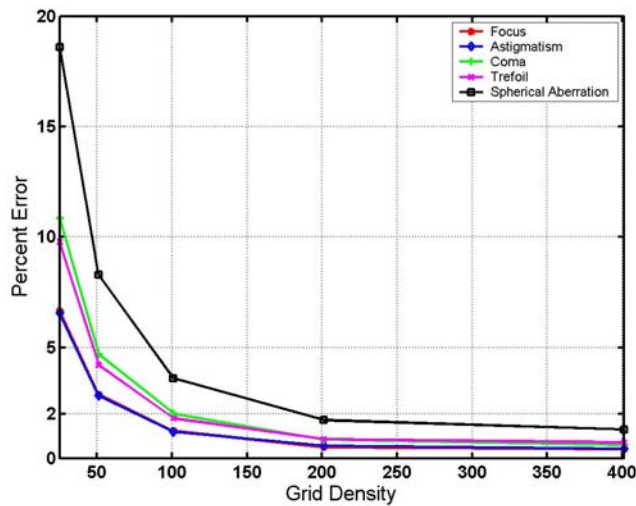Figure 4.4 Grid surface interferogram file percent error relative to Zernike interferogram files for $f$/1.0.



Figure 4.5 Grid Sag surface percent error relative to exact for $f$/1.0.

## 4.3 Thermal Soak Errors: Nonlinear Sag vs. Linear Sag

The Zernike polynomial surface definition using nonlinear and linear sag displacements were compared as a function of $f$/# in representing FEA surface displacements due to a thermal soak of 50-deg C. The surface was assumed fixed at the vertex with CTE of 23.6 ppm/C. Despace rigid-body errors were removed from the surface description and included in the optical model as a rigid-body perturbation. The nonlinear and linear sag displacements were fit to 10th-order Standard Zernike polynomials which provided an accurate fit to within 0.01%. The rms spot size results for $f$/1 - $f$/5 parabolas are listed in Table 4.5. The exact results were computed by changing the radius of curvature of the parabola directly in the optical model. The nonlinear sag computation duplicates the exact results while the linear sag computation is shown to be accurate to within 0.125%. For $f$/#'s smaller than $f$/1, the percent error between the linear and nonlinear sag displacements increases. As an additional exercise, an $f$/0.125 surface was modeled showing a 1% error using the linear sag displacements. This study was repeated for a 25-deg C thermal soak and the variation

between the linear and nonlinear sag displacements was approximately half indicating that over these temperature ranges the variation is linear.

| | $f/1$ | $f/1.5$ | $f/2$ | $f/3$ | $f/5$ |
|---|---|---|---|---|---|
| **Exact** | 1.0440E-02 | 1.0192E-02 | 1.0108E-02 | 1.0049E-02 | 1.0019E-02 |
| **Nonlinear Sag** | 1.0440E-02 | 1.0192E-02 | 1.0108E-02 | 1.0049E-02 | 1.0019E-02 |
| **Linear Sag** | 1.0453E-02 | 1.0204E-02 | 1.0120E-02 | 1.0061E-02 | 1.0031E-02 |
| **% Error** | 0.13 | 0.12 | 0.12 | 0.12 | 0.12 |

Table 4.5 Rms spot size predictions as a function of *f*-number.

## 5. EXAMPLE CASE

In general, when representing FEA surface displacements in an optical model, the exact solution and most accurate modeling method is not known. This section compares the rms spot size using the optical modeling methods previously discussed on a 12-inch *f*/1.5 parabolic lightweight mirror (clear aperture of 10.8-inches) mounted using three bipods. The optical substrate was subject to gravity acting along the optical axis and a thermal soak of 25-deg C. The finite element model and contour plots of the surface displacements for both gravity and thermal soak are shown in Figure 5.1. The dominant Zernike surface error for gravity is trefoil as compared to focus and spherical aberration for the thermal soak case. The rms spot size was computed using CODEV's and ZEMAX's Zernike polynomial surface definition with both nonlinear and linear sag displacements. CODEV surface interferogram files were created using both Zernike (231-term) and grid format (401 x 401). The ZEMAX's grid sag surface used an array size of 401 x 401. The 66-term Standard Zernike polynomial using CODEV's Zernike polynomial definition represented 97.0% and 93.7% of the surface displacements for the gravity and thermal soak cases, respectively. The 231-term Standard Zernike polynomial used in ZEMAX's Zernike Standard Sag surface represented 98.8% and 96.5% of the surface displacements for the gravity and thermal soak cases, respectively. Since the sets of Zernike polynomials do not exactly represent the sag displacements for either load case, the exact rms spot size is unknown. The rms spot size predicted by these modeling methods is listed in Table 5.1.
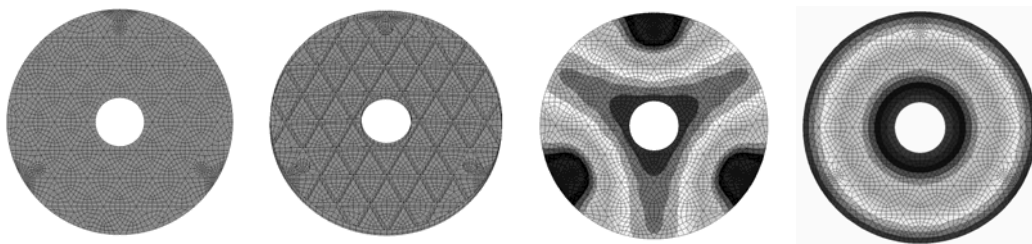


Figure 5.1  Front and rear view of FEA mirror model and FEA contour plots of gravity and thermal soak surface sag displacements.

It is expected that the most accurate modeling method based on the case studies is either the Zemax Standard Zernike surface (more Zernike terms than CODEV) or the Zemax Grid Sag surface. The Grid Sag surface has the benefit of representing the residual surface error not fit by the Zernike terms (this is also true for the grid interferogram file). In lieu of no exact value, the percent error of the various modeling methods relative to the ZEMAX Standard Zernike surface using nonlinear displacements is listed in Table 5.2. Each of the modeling techniques used to represent the gravity displacements have a percent error of less than 1%. The variation between the Standard Zernike Surface and the Grid Sag surface is 0.1%. The Zernike surface interferogram file (231-terms) represents the trefoil error due gravity within 0.5%. This is consistent with the case study results. For the thermal soak case, the difference between the Standard Zernike surface and the

Grid Sag surface is 1.1%.  The Zernike interferogram file has a percent difference of ~1.4% consistent with the case study prediction for surface errors dominated by focus and spherical aberration.  The percent error of the grid surface interferogram file is over 5%.  Minor differences in both sets of results are seen using the linear and nonlinear sag displacements for the thermal soak case.  As expected with no significant radial displacement under gravity, the variation between the linear and nonlinear sag displacements in negligible.

| | CODEV Sag Nonlinear | CODEV Sag Linear | ZEMAX Sag Nonlinear | ZEMAX Sag Linear | ZEMAX Grid Sag | Zernike INT Surface Normal | Zernike Grid Surface Normal |
|---|---|---|---|---|---|---|---|
| Gravity | 3.6844E-05 | 3.6844E-05 | 3.7038E-05 | 3.7038E-05 | 3.7073E-05 | 3.7193E-05 | 3.7249E-05 |
| Thermal Soak | 1.9746E-03 | 1.9751E-03 | 1.9920E-03 | 1.9926E-03 | 2.0138E-03 | 2.0196E-03 | 2.0990E-03 |

Table 5.1 RMS Spot Size using various optical modeling methods.

| | CODEV Sag Nonlinear | CODEV Sag Linear | ZEMAX Sag Nonlinear | ZEMAX Sag Linear | ZEMAX Grid Sag | Zernike INT Surface Normal | Zernike Grid Surface Normal |
|---|---|---|---|---|---|---|---|
| Gravity | 0.52 | 0.52 | 0.00 | 0.00 | 0.10 | 0.42 | 0.57 |
| Thermal Soak | 0.87 | 0.85 | 0.00 | 0.03 | 1.10 | 1.39 | 5.37 |

Table 5.2 Percent error of optical modeling methods relative to Zemax nonlinear sag.

## 5. SUMMARY

There exist several commonly used modeling options using commercial optical design software to represent finite element derived surface displacements.  Engineering judgment determines the 'best' modeling approach for a specific application and is dependent on the optical system, the desired accuracy, and the preferred manner in defining the surface errors in the optical model.  The intent of this paper was to explore the accuracy of these techniques for common surface errors and form guidelines to help the user determine the appropriate technique for a given application.  Note that mechanical modeling errors and uncertainties such as material properties, boundary conditions, loading conditions along with optical modeling errors and the effects of diffraction enter into the decision as to the most applicable approach.

1) **Rigid body motion:** Rigid-body errors, as a general recommendation, should be removed from the set of FEA surface displacements and represented in the optical model using tilts and decenters.  This is not only the most accurate approach as demonstrated in the case studies but it also allows optical degradation to be individually evaluated for each of rigid-body errors and the elastic surface deformations. Only use of the nonlinear sag displacements yielded accurate results when rigid-body errors were kept in the surface description.  Significant errors resulted when using surface normal displacements to represent rigid-body errors on the *f*/1 surface.

2) **Sag vs. Normal:** Both the polynomial and the rectangular array form of the surface interferogram file using surface normal displacements are less accurate than their sag counterparts.  This error increases with smaller *f*/#, becoming significant for *f*/#'s below *f*/1.  One advantage of using interferogram files is that they may be scaled.  The user may want to trade off some loss of accuracy in using interferogram files if interferogram files are the preferred approach in representing surface displacements.

3) **Polynomial vs. Array:**  This is entirely problem dependent.  If the polynomials chosen (type and number) are a good fit to the data (i.e. have a small residual error), they are the most convenient and compact form to use.  To improve the polynomial fit, the user may increase the number of terms up to the optical program's limit. In cases where the polynomials are not a good fit, such as local mount effects and quilting, a rectangular array may be used.  The accuracy of the array depends on the number of data points in the array and the surface deformation to be described.  It is recommended an array size of at least 401x401 be used.

The interpolation from an FEA mesh to a rectangular array is more accurate using cubic interpolation (over linear interpolation), if nodal rotations are generated in the FEA code.

4) **<u>Linear vs. Nonlinear Sag:</u>**  Since this is a function of the elastic radial displacement, there is negligible difference for most mechanical loads such as static gravity or vibrations.  For thermal loads, the radial displacement is a dominant effect and must be taken into account.  The nonlinear fit is more accurate, but the linear fit offers the advantage of scalability.

5) **<u>In General:</u>** The most accurate means to represent elastic surface errors is to use the Zernike polynomial surface definition based on the sag component of surface displacement (CODEV's SPS ZRN or ZEMAX Standard Zernike Sag). If a perfect polynomial fit is achieved to the FEA surface displacements, use of the nonlinear sag displacements yields exact results.  Uniform arrays of data are useful to represent displacements when an accurate polynomial fit is unable to be achieved.  The ZEMAX Grid Sag Surface utilizes sag displacements and thus is superior to CODEV's surface interferogram files using surface normal displacements for powered surfaces.

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