Design strength of optical glass

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ABSTRACT

Brittle materials such as glass do not possess a single characteristic strength. The strength of the material is dependent on the distribution of cracks or surface flaws. These factors, coupled with the inherent brittleness (cause of catastrophic or rapid failure) mean that extremely conservative design approaches are typically used for optical elements made of glass. Determining a design allowable for glass elements is critical for optical systems using relatively brittle glass types or for optical elements subject to relatively high stress levels. Rule-of-thumb tensile design strengths are typically at 1000 - 1500 psi for nominal glass materials. This neglects the specific glass composition, subcritical crack growth, surface area under stress, and nature of the load - static or cyclic. Several methods to characterize the strength of optical glass are discussed to aid engineers in predicting a design strength for a given surface finish, glass type, and environment. These include estimating fracture toughness for a given glass, predicting inert strength using material test data, and lifetime predictions accounting for static fatigue and cyclic loading. Determining a design strength for a spaceborne optical element is discussed.

Keywords: fracture, static fatigue, stress corrosion, dynamic fatigue, fracture toughness, time-to-failure

1. INTRODUCTION

The strength of glass is governed by the random distribution of the size, orientation, and location of the surface flaws in relation to regions under stress. The surface flaws are commonly created during the grinding and finishing operations of the optical substrate. Due to the scatter in the flaw size, the strength of glass is a function of the size of the glass component. Hence, there is no deterministic strength for brittle materials (unless the flaws are extremely uniform). These flaws or cracks propagate under tensile loads to a critical value and then experience uncontrolled crack growth until the part physically fractures.

A key in maximizing the strength of a glass component is to minimize the surface flaws during the manufacturing process. This is commonly achieved through a controlled grinding and polishing process that aims to decrease the depth of the surface flaws after each step of the operation. Removal rates are designed to remove a depth of material that is equal to or greater than the maximum flaw depth[1]. The maximum flaw depth may be approximated as three times the average diameter of the grinding particle. By using special finishing techniques, very high strengths in glass may be obtained. For example, chemical polishing with hydrofluoric acid have produced tensile strengths over a 1,000,000 psi. Typically, however, reducing the flaw depth below 0.001" is difficult using conventional techniques.

For glass components subject to stress levels that exceed a 1000 psi or when using brittle glasses characterized by comparatively low fracture toughness, material strength testing is necessary to characterize

the strength properties of the material. Material testing enables the strength to be characterized using a statistical distribution (most common is the Weibull distribution) and determines the rate at which cracks grow within a material. This data is then used to develop a design strength and predict the time to failure.

2. FRACTURE TOUGHNESS

A fracture mechanics approach to the failure of brittle materials involves the growth of surface flaws or cracks under tensile loads. The resistance to flaw growth in a material is known as the fracture toughness or the critical stress intensity factor, K_{IC}. A comparison of the fracture toughness of several glass types are shown in Table 2.1. Failure occurs when the stress intensity factor, K_I, which describes the level or intensity of the stress distribution just ahead of the crack tip, exceeds the fracture toughness of the material,

$$K_I > K_{IC}. \tag{2.1}$$

The stress at the tip of the crack is concentrated due to the crack geometry and the lack of plastic deformation. When the stress value exceeds the strength of the bonds in the glass molecular network, crack propagation occurs. The stress intensity may be computed for a given crack size and geometry using Griffith's law expressed below

$$K_I = Y\sigma\sqrt{a} \tag{2.2}$$

where Y is a crack geometry factor, σ is the nominal stress, and a is the flaw size. A design strength may be estimated based on an assumed flaw size with an appropriate factor of safety.

Fracture toughness data is not always available for the glass types in a given optical instrument. However, the fracture toughness for common optical glasses (Schott, Ohara, Hoya) may be determined using the manufacturers published data on lapping hardness[2]. Lapping hardness is defined as the volume of material removed under a predefined set of processing conditions including pressure, velocity, coolant, and abrasive. Rates of removal, surface roughness, and subsurface damage are a function of a glass's mechanical properties including the elastic modulus, hardness, and fracture toughness. Lapping hardness may be derived from the mechanical properties controlling elastic deformation (elastic modulus, E), resistance to flow (hardness, H_k), and resistance to cracking (fracture toughness, K_{Ic}) through a combined figure of merit given by the following

$$E^{7/6} / \left[K_{Ic} H_k^{23/12}\right]. \tag{2.3}$$

The glass manufacturers each publish data on the lapping hardness of their glasses; Schott expresses this data as a height reduction, Δh - (see Schott Technical Information Bulletin No. 4), Ohara as an Ohara factor, Aa, and Hoya as a Hoya factor, FA. Using this data and the curves that have been published showing the relationship between the above figure of merit and lapping hardness, fracture toughness may be estimated.

3. STATIC FATIGUE

A phenomenon that complicates the computation of a design strength is that glass materials are susceptible to subcritical crack growth in monotonic tension due to the influence of moisture e.g. under a constant state of stress, the size of the crack increases until the stress intensity at the crack tip exceeds the fracture toughness and failure occurs. This is known as stress corrosion and is a form of static fatigue. (In this definition,

fatigue refers to the degradation of mechanical properties with time. Later dynamic fatigue, referring to the effects of cyclic mechanical loading, is discussed.) Stress corrosion is due to the reaction of the glass material with water at the crack tip. Even small amounts of water vapor can markedly reduce the lifetime of the component under static load conditions. Thus the strength of glass is a function of the humidity level and the time in which the component has been loaded. Generally, the more moisture that is present, the greater the reaction resulting in greater crack growth. Stress corrosion and the effects of moisture may be measured by loading test specimens under a constant stress and using various moisture levels and recording the time to failure. These are known as

Fracture Toughness, K_{IC} psi $\sqrt{in}$	
Fused Silica	674
BK7	774
SF5	519
SK16	710
LaK10	865
F2	500
SF58	346

Table 2.1 Variation in Glass fracture toughness.

time-to-failure studies. For a constant moisture level, the time-to-failure is typically shown to decrease as the stress in increased. Fracture strength versus load duration curves may be determined by varying the load duration. These curves are often normalized using the inert strength of the glass material - inert strength refers to the strength *sans* the effects of stress corrosion. This is achieved by measuring the strength of test specimens cooled to temperatures of liquid nitrogen where the effects of water are negated.

Time-to-failure studies are often time consuming and an accelerated method known as dynamic fatigue testing is often employed. In this case, the stress applied to the test specimen is not constant but applied at a constant rate. When the stress rate is fast, there is no time for crack growth and the specimen fails at a high stress. Conversely, when the stress is applied slowly, the crack has an opportunity to propagate and the specimen fails at a lower stress. The average stress at failure is often plotted versus the rate of stress using a log-log scale. The slope of the line characterizes the fatigue resistance of the glass and is known as the fatigue resistance parameter.

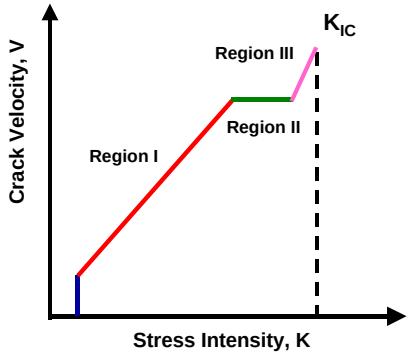

Figure 3.1 Regions of a typical V-K curve

Crack growth may also be studied using controlled growth of well-defined surface flaws. The crack velocity (V) is measured and plotted versus the stress intensity at the crack (K_I) on a semilog scale known as the V-K

curve. A schematic of a typical V-K curve for glass is shown in Figure 3.1[3]. The various modes of crack propagation are shown; crack propagation is initiated once the stress intensity at the crack tip exceeds a given threshold; once the threshold has been reached, crack propagation in region I is a function of the reaction of the surface flaw with moisture; region II represents a transition region where part of the crack region is reacting to moisture and part is not; unstable / uncontrolled crack growth occurs in region III, and water is unable to react with the crack due to the speed of propagation. Once the stress intensity reaches the fracture toughness of the material, K_{IC}, the part fractures. Region I is of primary interest since this is the region of stable crack growth. The slope of this line is another measure of the fatigue resistance parameter. Crack propagation in region I for several glasses, based on data from Weiderhorn[4], is shown in Figure 3.2.

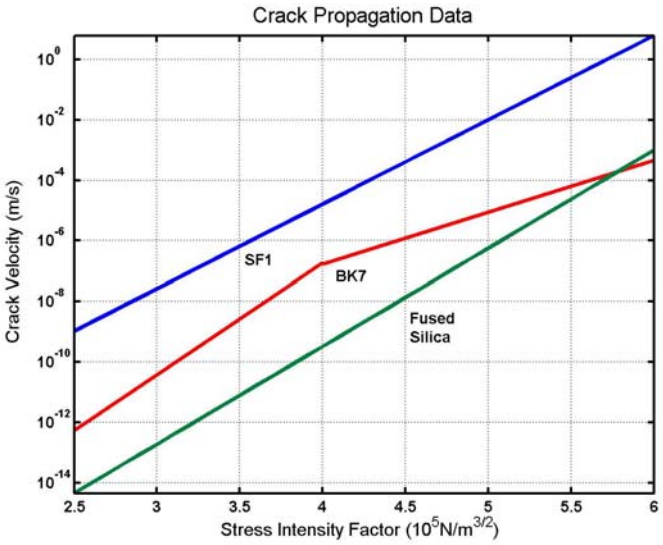

Figure 3.2 V-K Curves for SF1, BK7, and Fused Silica (Region 1)

It should be noted that for crack growth due to static fatigue, failure is considered when the stress intensity at the crack tip exceeds the value of the materials fracture toughness. However, for certain brittle solids it has been shown that as the crack size increases the stress intensity at the crack is reduced[5] . A material that experiences this phenomenon is known as toughened. Often an effective stress intensity is used such that the nominal stress intensity, K_I, is reduced by a shielding component, K_s, expressed as

$$K_{eff} = K_I - K_s \tag{3.1}$$

4. INERT STRENGTH AND TIME-TO-FAILURE

Predicting design strength provides the basis for which the optical components and mounting configuration may be designed such that failure does not occur over the operational lifetime of the instrument. Three approaches based on material strength testing of test specimens are discussed below. The first method determines the inert strength of the component as represented by a Weibull statistical distribution. In this approach, the statistical parameters representing the component are determined by scaling the test specimen statistical parameters based on the surface area under stress. A design strength may then be determined based on an acceptable probability of failure. This method assumes no crack growth occurs during the life of the component. The second approach is based on dynamic fatigue testing which is used to predict the lifetime of a component assuming subcritical crack growth occurs during the service life (i.e. stress corrosion). The

third method combines the previous two approaches and develops time-to-failure curves as a function of probability of failure.

4.1 Inert Strength

The strength and probability of failure of actual components may be extrapolated from test specimen data using Weibull probabilistic methods. This theory-based procedure assumes that the surface flaws that limit the strength is the same for the specimens as it is for the components and that the surface flaw population is unvarying with time in service (i.e. no static fatigue). The manner in which this is achieved is by fitting a two-parameter Weibull distribution to 30 or more test specimen strength data. The probability of failure for a given test specimen may be determined for an applied stress, σ, as expressed below

$$P_f = 1.0 - e^{\left[-\left(\frac{\sigma}{\sigma_o}\right)^m\right]} \tag{4.1}$$

where P_f is the probability of failure, σ_o is the characteristic strength (63.2% of the specimens fail at this stress level) and *m* is the Weibull modulus, an indicator of the scatter of the data. Once the strength of the glass has been statistically characterized, probability of failure versus tensile stress curves may be created. These curves allow a design strength to be selected based on a chosen probability of failure. The probability of failure versus tensile stress is shown for several Schott glasses in Figure 4.1. The data used in this figure is based on a double ring test using a surface area of 113 mm^2 at room temperature[6]. The surface of the glass specimens were polished using a loose silicon carbide grain with an average particle size of 9 um.

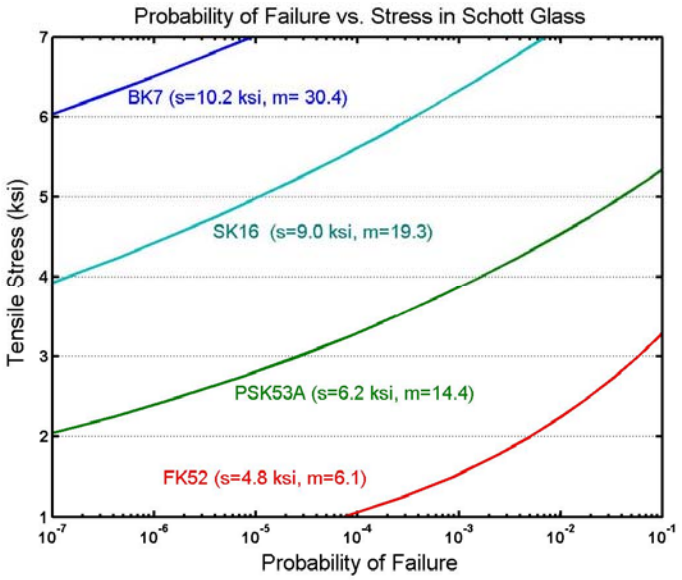

Figure 4.1 Probability of Failure Curve for Several Schott Glasses

Since the strength of glass is a function of the surface area that is under stress, the test specimen strength distribution may be used to derive the strength distribution of the actual glass component using a ratio of the surface areas under tensile stress. An effective surface area, A_{eff}, may be computed for a component with a varying stress field such as that computed via finite element analysis using the following relationship

$$A_{eff} = \int \left(\frac{\sigma}{\sigma_{\max}} \right)^m dA \tag{4.2}$$

where σ represents the surface tensile stress over area dA, and σ_{max} is the maximum surface tensile stress. The characteristic strength of the component is then computed by[7]

$$\frac{\sigma_o^{specimen}}{\sigma_o^{component}} = \left(\frac{A_{component}}{A_{specimen}} \right)^{\frac{1}{m}} \tag{4.3}$$

The above relationship allows the characteristic strength of the actual glass component to be expressed and hence the component strength is statistically characterized since the Weibull modulus is assumed constant. For test specimen data to be extrapolated to component geometries, the surface of the test specimens must be prepared exactly as the component. This ensures that the Weibull modulus used in the scaling law is constant between the test specimens and component. This general method, while not exact, offers many benefits as it is often impractical to test actual components in their true loading condition in sufficient quantity to yield reliable results. The method is adequate for multi-axial, tensile loaded specimens, provided that the second or third principal stresses are significantly less than the principal tensile stress. If this is not the case, then more sophisticated analyses that take into account the effect of multi-axial tensile stresses on flaws are required.

4.2 Lifetime & Time-to-Failure Predictions

A more detailed and comprehensive method to predict the lifetime of a glass component takes into account the reduced strength of the material over time due to subcritical crack growth. This requires knowledge of how cracks grow for a given material and may be obtained using the methods discussed in Section 3. From this data, analytical expressions exist to compute the total time-to-failure, t_s, for a component under a constant static stress with a known surface flaw as expressed below[8]

$$t_s \sim \frac{2}{\sigma^2 Y^2} \int_{K_{Ii}}^{K_{IC}} \left(\frac{K_I}{\mathrm{V}} \right) dK_I \tag{4.4}$$

where K_{Ii} is the initial stress intensity factor, and V is the crack velocity. For crack geometries consistent with those found on glass surfaces, and assuming an exponential from of the V-K curve,

$$V = v_o e^{\beta K} \tag{4.5}$$

where ν_o and β are constants, then the time-to-failure may be expressed as

$$t_s = \frac{2K_{Ii}}{\beta \sigma^2 Y^2 v_o e^{\beta K}} \tag{4.6}$$

The V-K curve is also commonly expressed as a power law,

$$V = AK^n \tag{4.7}$$

where A and N are constants. This yields a second expression for the time-to-failure[9]:

$$t_s = 2(K_{Ii}^{2-n} - K_{IC}^{2-n})/[(n-2)A\sigma^2 Y^2] \tag{4.8}$$

Using the above expressions, time-to-failure curves may be developed based on an initial flaw size. The initial flaw size may be estimated using the inert strength of the material from which an initial K_{Ii} is determined. Curves may be drawn for different flaw sizes as illustrated in Figure 4.2 or to compare different materials as shown in Figure 4.3.

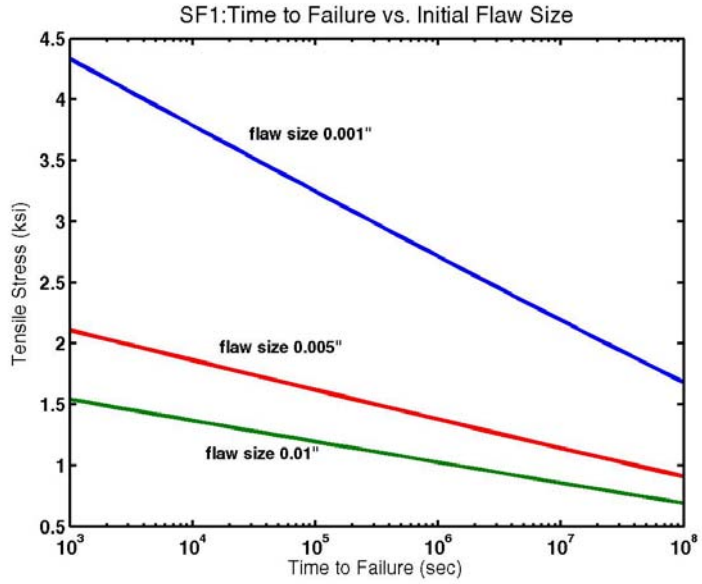

Figure 4.2 Time-to-Failure vs. Initial Flaw Size

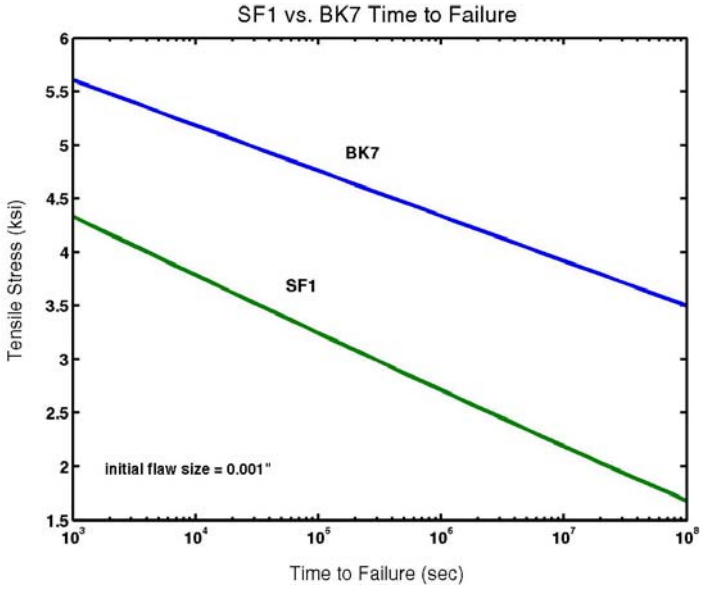

Figure 4.3 Schott Glass BK7 and SF1 Time-to-Failure Curves

4.3 Lifetime Prediction and Probability of Failure

Design strength diagrams may be created by coupling the methods discussed in Section 4.1 & 4.2. This yields time-to-failure versus tensile stress curves as a function of probability of failure. The initial flaw distribution as characterized by a Weibull distribution (Section 4.1), allows the initial stress-intensity at the crack tip to be estimated by,

$$K_{Ii} = K_{IC}\left(\frac{\sigma}{\sigma_{IC}}\right) \tag{4.9}$$

where σ_{IC} is the stress at fracture. Using the stress at fracture, σ_{IC}, for σ in equ. (4.1) and solving for σ_{IC}, and then substituting into equ. (4.9) yields the following relationship[10]:

$$K_{Ii} = K_{IC}\left(\frac{\sigma}{\sigma_o}\right)\left[\ln\left(\frac{1}{1-P_f}\right)\right]^{-1/m} \tag{4.10}$$

The initial stress intensity value is then used in the time-to-failure equations to develop a family of design curves. These design curves account for sub-critical crack growth and enable a design strength to be selected based on the required level of safety as shown in Figure 4.4.

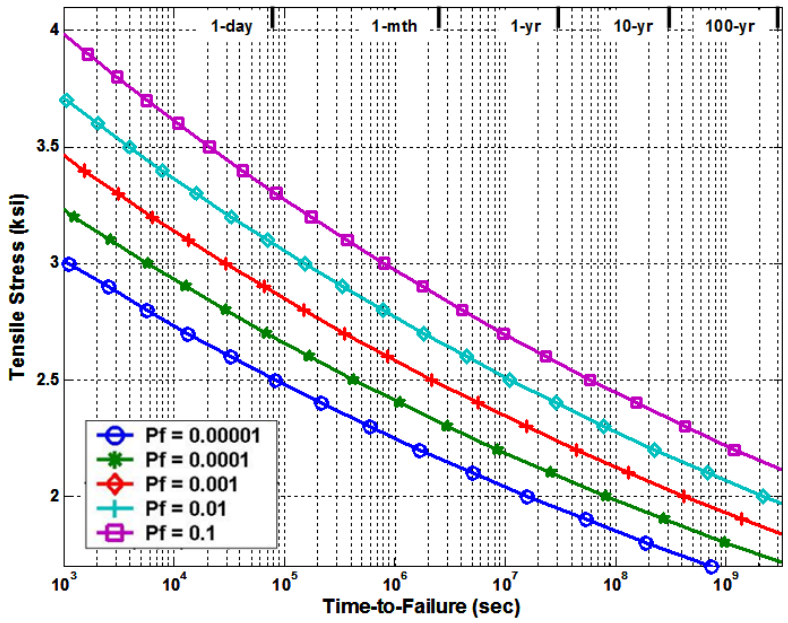

Figure 4.4 BK7 Design Strength Curve

The values used to compute the curves in Figure 4.4 were based on a Weibull characteristic strength of 10.2 ksi, a Weibull modulus of 30.4, a fatigue resistance parameter of 23.6, and a power law coefficient, A, of 128.

5. CYCLIC FATIGUE CRACK GROWTH

Prior to the recognition that brittle solids exhibit fatigue effects, a method to compute the time-to-failure due to cyclic loading as a function of the static crack growth curves was developed[11]. This approach assumes that the cyclic stress field may be represented as a summation of incremental static loads, the compressive nature of the load has no effect on crack growth, and the static and cyclic failure mechanisms are equivalent.

The time-to-failure due to cyclic loading, t_c, is expressed below

$$t_c \cong g^{-1}\left(\frac{\sigma_s}{\sigma_a}\right)^n t_s \tag{5.1}$$

where t_s is the time-to-failure under a static stress, σ_s is the static stress, and σ_a is the average cyclic stress. The factor g^{-1} is given analytically and graphically for various cyclic loads as a function of the average stress and the amplitude of the cyclic load. For the special case of a sinusoidally varying load with an average stress of zero, g^{-1} is given as

$$g^{-1} = \sqrt{2\pi n}\left[1 + \frac{1}{4n} + \frac{1}{32n^2} + ...\right]. \tag{5.2}$$

Thus, design strength curves may be developed such as those in Figure 4.4 accounting for cyclic loading.

However, it has been shown in the past several decades that crack growth under cyclic mechanical loads (fatigue crack growth) for brittle materials is much more complicated than crack growth under static load conditions. Identifying actual fatigue effects in brittle solids as compared to their metal counterparts didn't come to acceptance in the community until the late 1970's. Fracture surfaces for brittle solids due to static and cyclic loads are very similar as contrasted to the dislocation activity of metals whereby methods of fractography are able to identify the failure mechanism by interpreting the failed surfaces i.e. brittle solids experience relatively little plasticity. This coupled with the high precision and expensive testing along with the need to statistically characterize the behavior due to the high degree of scatter in the results, helps explain why the fatigue behavior in brittle solids lags significantly behind the knowledge of fatigue behavior of metals.

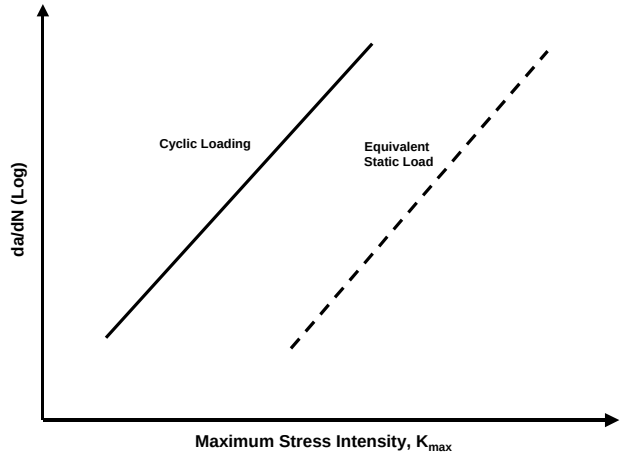

Figure 5.1 Crack growth comparison of cyclic loading to equivalent static loading per cycle

A true fatigue effect has been identified in brittle solids leading to a decelerated or accelerated crack growth under cyclic loads as compared to static loads. It has been shown that during repeated tension-compression cyclic loading whereby the crack surfaces are repeatedly placed in physical in contact during the compression stroke of the cycle, that crack growth may be accelerated[12]. See Figure 5.1. Conversely, crack growth may be decelerated by such factors such as debris particles wedging between the contact surfaces. The beneficial and detrimental effects that cyclic loading have on time-to-failure is shown in Figure 5.2.
An additional note regarding static and cyclic loads is that fatigue in metal materials is governed by the range of the stress intensity, ΔK, achieved during cyclic loading as given by the Paris equation:

$$\frac{da}{dN} = C(\Delta K)^m \tag{5.3}$$

where da/dN is the crack propogation rate per fatigue cycle. For brittle materials, a modified Paris Law has been proposed for cyclic crack growth that expresses crack growth as a function of $K_{max,}$ the maximum stress intensity factor, and ΔK, the difference between the maximum and minimum stress intensity during a cyclic load, given below[13]:

$$\frac{da}{dN} = B(K_{\max})^g(\Delta K)^q \tag{5.4}$$

where g is known as the static crack growth exponent and q as the cyclic crack growth exponent. It has been shown for several brittle solids that g is typically much larger than q indicating that crack growth depends primarily on the maximum value of the stress intensity factor, K_{max}, and not on ΔK. These studies have helped show that the mechanisms responsible for fracture under static and cyclic loading are considered the same.

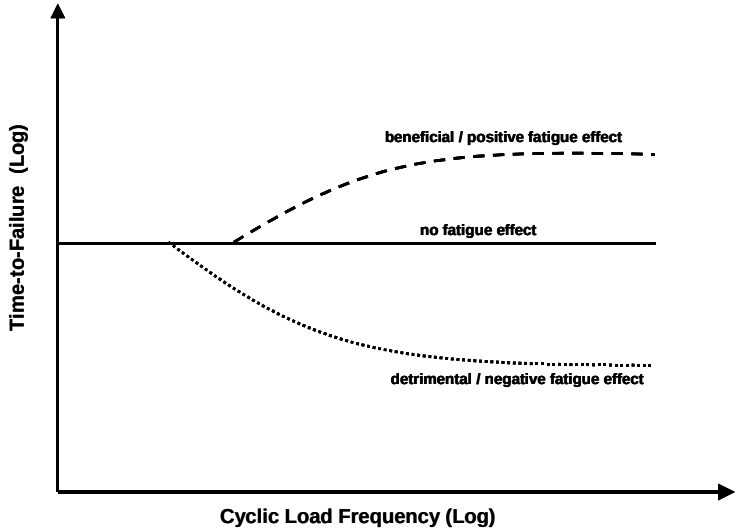

Figure 5.2 Beneficial and detrimental effects of cyclic loading on time-to-failure

6. DESIGN STRENGTH EXAMPLE

The above methodology was applied to determine a design strength for a BK7 Schmidt corrector plate as part of a Schmidt Telescope assembly. The corrector plate is 10-inches in diameter, 0.75-inches thick, and mounted within an aluminum ring frame with six titanium tangential flexures bonded to the edge of the optic.

A Weibull characteristic strength of the corrector plate was computed by scaling the test specimen data (characteristic strength of 10.2 ksi, Weibull modulus of 30.4) using an effective surface area based on the surface tensile stress distribution. This yielded a characteristic strength of the corrector plate of 9.7 ksi. Peak corrector plate stresses occur where the adhesive bond attaches the flexure to the corrector plate. The component design strength coupled with the crack growth data, represented by a power function (A = 23.6; N = 128), yielded the design curves in Figure 6.1. A design strength was then selected of 1.7 ksi based on a probability of failure of 10^{-5} and a desired survival time of 10-years. This analysis made the assumption that the moisture present during the collection of the crack growth data is the same as that during operation in orbit, which is a conservative assumption. Thus the actual time-to-failure may be greater.

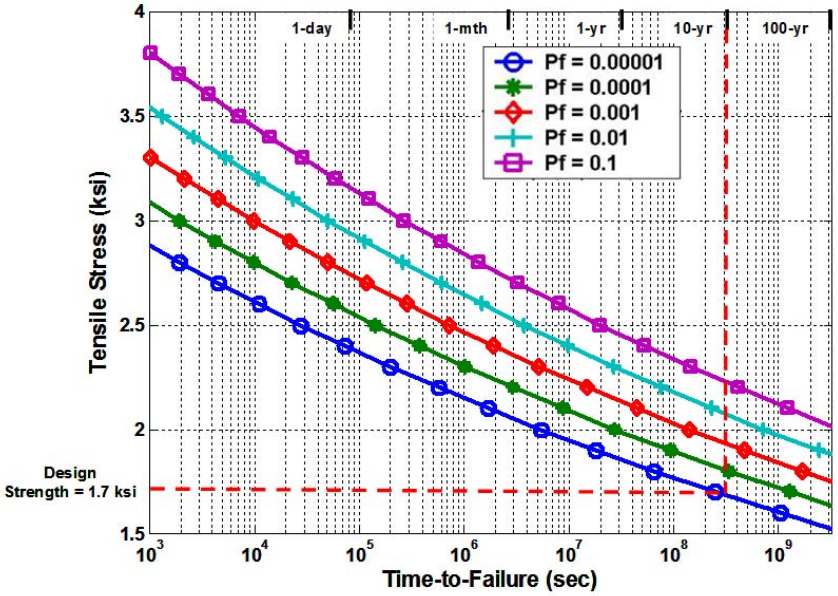

Figure 6.1 Corrector Design Strength Curves

SUMMARY

Determining a design strength for glass optical elements is critical when stress levels are expected to exceed 1000 psi in common optical glasses or when comparatively brittle glass materials are employed. In addition, a design strength enables optical mounting methods to be selected that are consistent with the required factors of safety. The strength of optical glass is governed by the surface finish, surface area under tensile stress, glass composition, and the type of loading. Knowing the fracture toughness of the material allows a design strength to be based upon an estimated flaw size. The flaw size may be estimated based upon the size of the grinding particles. The design strength of a material may also be based upon probabilistic methods using a Weibull statistical distribution. Both the above methods assume no crack growth occurs during the lifetime of the part. For glass materials exposed to stress levels over a significant time, subcritical crack growth occurs that decreases the strength of the glass. Crack growth data may be determined for a given material using material test methods that allow the time-to-failure to be predicted. Coupling this with statistical data on the inert strength of the material enable a family of design curves to be developed. For glass elements subject to dynamic or cyclic stress levels, predicting the strength of the glass is more complicated. Fatigue effects have been identified in brittle solids that may increase or decrease crack growth as compared to a static load. In lieu of crack growth data during cyclic loading, conservative estimates of glass strength are recommended.

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