Making FEA Results Useful in Optical Analysis

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ABSTRACT

Thermal and structural output from general purpose finite element and finite difference programs is not in a form useful for optical analysis software. Temperatures, displacements and stresses at arbitrarily located FE nodes can not be input directly into optical software. This paper discusses the post-processing steps required to present the FE data in a useable format. Specific issues include optical surface deformations, thermo-optic effects, adaptive optics, optimization, and dynamic response. Finite element computed optical surface deformations are fit to several polynomial types including Zernikes, aspheric, and XY polynomials. Higher frequency deformations are interpolated to a user-defined uniform grid size using linear, quadratic, or cubic finite element shape functions to create interferogram files. Three-dimensional shape function interpolation is used to create OPD maps due to thermo-optic effects (dn/dT), which are subsequently fit to polynomials and/or interferogram files. Similar techniques are also used for stress birefringence effects. Adaptive optics uses influence functions to minimize surface error before or after pointing and focus correction. A dynamic analysis interface allows optical surface perturbations (rigid-body motions, elastic surface deformations) to be calculated for transient, harmonic and random response.

Keywords: optomechanics, finite element analysis, optical design, Zernike polynomials, interferogram files, dynamic analysis, optimization, thermo-optic effects, stress birefringence

1.0 THERMAL-STRUCTURAL-OPTICAL ANALYSIS

1.1 Performance Predictions

In the modern design of optical systems, accurate performance predictions are required. To accomplish this, the system must be analyzed for thermal and structural response and the resulting optical behavior predicted. For instance, a lens system may have significant thermal gradients which can be predicted in a thermal analysis code such as Thermal Desktop. The thermoelastic deflections and stresses can be then predicted in a structural finite element program such as Nastran, if the temperature data can be written at the structural nodes. Typically, thermal and structural meshes are different. In order to understand the optical performance, these temperatures, structural deformations, and stresses must be transferred to an optical analysis program for a detailed system level performance. Optical programs such as CodeV cannot accept data at arbitrary meshes used in a finite element model, so the data must be transferred to a useable form.

1.2 Interface Issues

Each discipline in Figure 1 has it's own suite of tools and modeling techniques. The difficulty has always been in getting data from one discipline to another in an accurate and efficient manner. The developers of major computer codes such as Nastran, Thermal Desktop, and CodeV have many other demands placed on them which limits the amount they can spend on the interface to other specialized tools. To accomplish the transfer of data, one must deal with different coordinate systems, sign conventions, units, format, and syntax. This paper presents a commercially available computer tool SigFit[1] for transferring data from thermal and structural analyses to optical analysis programs.

Integrated Opto-Mechanical Analysis

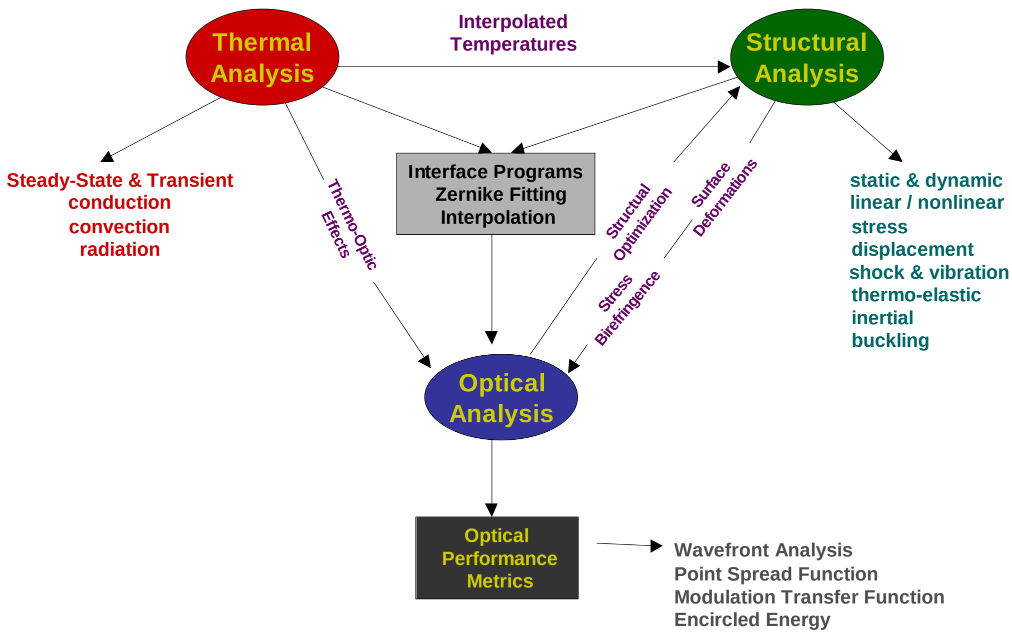

Figure 1. Integrated Analysis

2.0 THERMAL ANALYSIS TO OPTICAL ANALYSIS

2.1 dn/dT Effects

The change of index (n) with temperature (T) in a lens is referred to as the dn/dT effect. As the temperature changes (ΔT) along an optical beam length (L), an optical path length difference (OPD) is created.

$$OPD = \frac{dn}{dT}(\Delta T)L \qquad (2.1)$$

Optical analysis programs can often accept uniform temperature changes or linear gradients as input which may satisfy first order analyses. For a more accurate analysis of a lens with an arbitrary thermal loading, a numerical solution is often required. Both finite element (FE) and finite difference (FD) heat transfer analyses provide temperatures at discrete nodal locations throughout an optic. Converting this nodal data to a form which can be passed to an optical program requires a specialized tool like SigFit[1].

In SigFit, equation 2.1 is numerically integrated through the optic to create an OPD wavefront map which may be output as a Zernike Polynomial table or an interferogram array in CodeV or Zemax format. A 3D solid finite element model of the optic and the nodal temperatures are required as input. The steps within SigFit are:

1) Read the model, temperatures, dn/dT data, entrance (R_i) and exit (R_o) aperture radii
2) From each node on the entrance surface, pass an average ray through the optic
3) For each integration step along the ray, use 3D interpolation[2] to find the temperature
4) Numerically integrate and sum the OPD for each ray.
5) Fit Zernike polynomials[3] to the OPD over the surface and write coefficients in optical format

6) Optionally, interpolate from FE mesh to regular interferogram array and output in optical format
7) Write graphical (nodal) files for viewing the OPD results

2.2 dn/dT Example

A single lens shown in Figure 1 has a laser load applied causing the temperature profile in Figure 2. The optical aperture is 7/8 of the OD, so the outside elements are not used in the optical calculations. The integrated average temperature through the thickness is shown in Figure 3 and the resulting OPD caused by the index change is shown in Figure 4. When processed in SigFit, the Zernike polynomial fitting results to the OPD are shown in Table 1 or in ORA/CodeV format in Table 2.

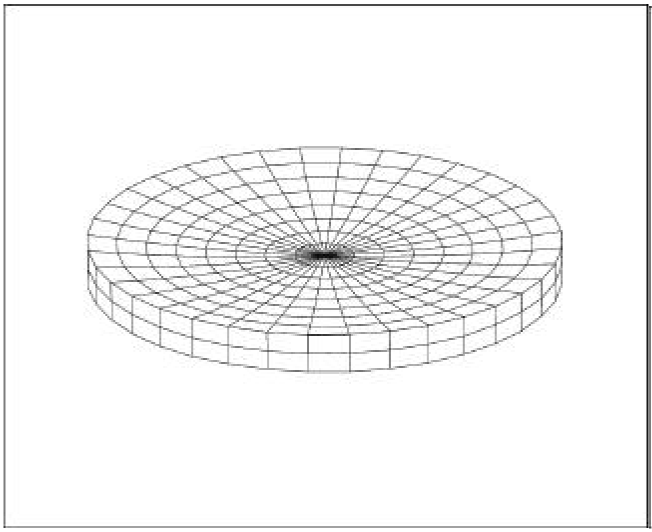

Figure 1. Lens Finite Element Model

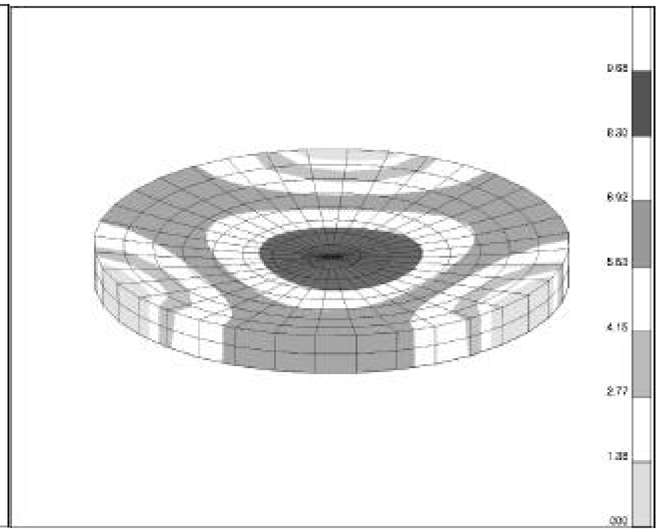

Figure 2. Temperature Profile

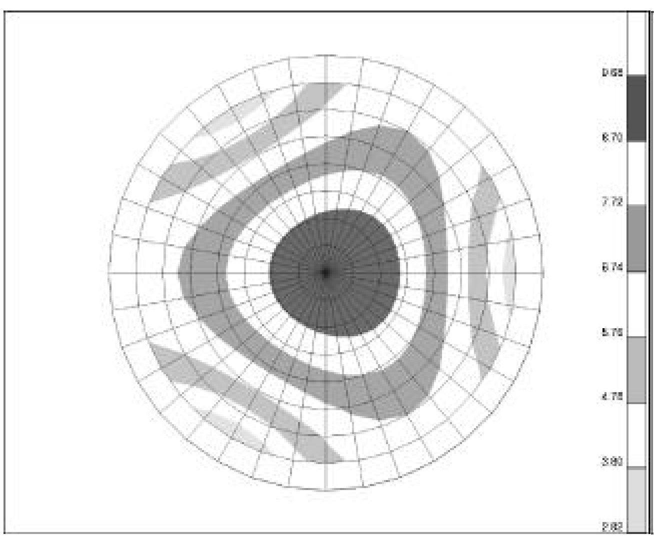

Figure 3. Average Temperature through Thickness

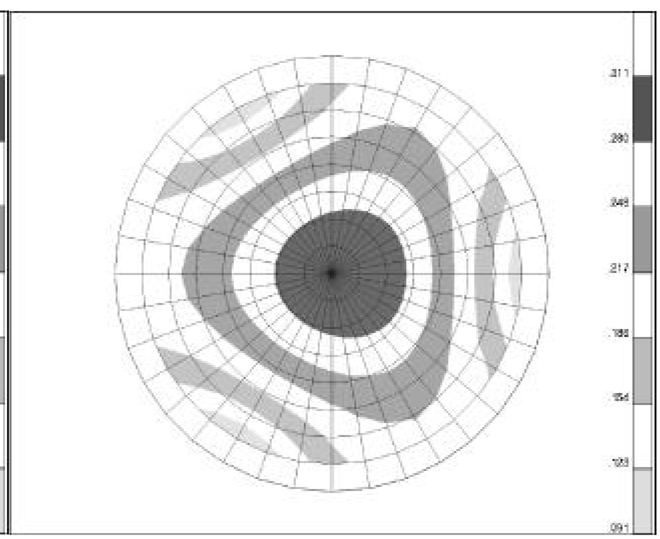

Figure 4. OPD resulting from dn/dT effects

Table 1. Zernike Coefficient fit to dn/dT OPD

```
================================================================================
Sigmadyne, Inc.       SigFit  Version=v2002-r2
--------------------------------------------------------------------------------
dn/dT OPD Wavefront: Best-Fit Zernike Polynomial Coefficients

   Surface  =  1     FE units  =      in          Wavelength= 2.4910E-05
```

Order K	N	M	Aberration	Magnitude (Waves)	Phase (Deg)	Residual RMS	Residual P-V
			Input(wrt zero)			.2253	.2193
1	0	0	Bias	.20942	.0	.0483	.2193
2	1	1	Tilt	.00752	0.0	.0482	.2226
3	2	0	Power (Defocus)	-.07666	.0	.0221	.1222
4	2	2	Pri Astigmatism	.00048	0.0	.0221	.1222
5	3	1	Pri Coma	.00486	180.0	.0220	.1183
6	3	3	Pri Trefoil	.07293	-60.0	.0093	.0379
7	4	0	Pri Spherical	.01821	.0	.0043	.0297
8	4	2	Sec Astigmatism	.00077	90.0	.0043	.0293
9	4	4	Pri Tetrafoil	.00069	-45.0	.0043	.0293
10	5	1	Sec Coma	.00561	0.0	.0040	.0290
11	5	3	Sec Trefoil	.00203	60.0	.0040	.0287
12	5	5	Pri Pentafoil	.00037	-36.0	.0040	.0285
13	6	0	Sec Spherical	-.00459	.0	.0037	.0285
14	6	2	Ter Astigmatism	.00257	0.0	.0036	.0275
15	6	4	Sec Tetrafoil	.00005	0.0	.0036	.0275
16	6	6	Pri Hexafoil	.02020	-30.0	.0010	.0064

Table 2. ORA/CodeV OPD input as Zernike Coefficients

```
!================================================================================
! Sigmadyne, Inc.       SigFit  Version=v2002-r2
! -------------------------------------------------------------------------------
S01 L01
ZRN 28 WFR SSZ 1.0 WVL   6.32714E-01
   2.09422E-01  7.52110E-03 -7.11628E-08  4.79822E-04 -7.66609E-02 -1.47388E-08
  -7.29272E-02 -4.85886E-03  5.47452E-08 -2.55165E-06 -6.92389E-04 -7.65609E-04
   1.82062E-02  1.14341E-07 -1.97729E-08 -3.70589E-04 -2.03251E-03  5.60957E-03
   1.99481E-07  2.71779E-06 -3.24532E-08 -2.01992E-02  4.50456E-05  2.56709E-03
  -4.58987E-03  1.61060E-07  5.92139E-08 -2.52364E-06
```

For a more detailed example of a multi-lens system, see reference 4 or 5.

3.0 THERMAL ANALYSIS TO STRUCTURAL ANALYSIS

3.1 Issues

When a single FE model is used for both thermal and structural analyses, the transfer of temperature data is automatic. In many cases, the thermal analyst uses a different mesh size (typically coarser), than the structural analyst. In order to provide temperatures at the structural nodes, some form of interpolation is required. When dealing with FE data, 3D shape function interpolation[2] is highly accurate. Other interpolation techniques such as closest node, nodal averaging, or conduction models produce an irregular temperature field which creates inaccurate thermoelastic effects.

3.2 Tools

A highly robust commercial tool for thermal analysis is CRT/Thermal Desktop[6] in which an analyst can use both finite element and finite difference entities as well as more complex geometrical surfaces to describe a system model. The resulting temperature profiles can then be interpolated on to a structural mesh in MSC/Nastran format.

Any graphics program which can contour temperatures does an interpolation to create the graphics. MSC/Patran uses this “field”capability to interpolate temperatures from one mesh to another.

For the example lens above, the thermal and structural models were identical, so no interpolation was necessary.

4.0 STRUCTURAL ANALYSIS TO OPTICAL ANALYSIS

4.1 Surface Distortions

The structural effects of surface distortion are calculated in an FE code must be passed to an optical program. Again, the displacements are calculated at the arbitrary FE mesh nodes which the optical codes cannot accept. Within SigFit, the surface displacements are processed and fit[3] with a variety of polynomials (Zernike, Aspheric, X-Y) which can be output in ORA/CodeV or Focus/Zemax format. There are several issues and options which must be addressed in the fitting process to provide correct data for optical analysis, including:

1) Coordinate system location and orientation; left or right hand rotation angles
2) Surface sign convention (bump=+/-)
3) Surface displacements as axial sag (dz) or surface normal (dn)
4) Surface sag correction for radial growth[7]
5) Aperture and obstructions
6) Nodal area weighting[8]
7) Zernike polynomial ordering and normalization[8]
8) Units in the FE model verses the optics model
9) Surface numbering scheme

Within SigFit each of the above issues are addressed to convert structural data to optical data. For the lens in Figure 1, surface 1 is located on the convex surface on the bottom. The thermo-elastic distortion due to the temperatures in Figure 2 is shown in Figure 5 for surface 1 after correction for optical sign convention and radial growth. The resulting Zernike fit is given in Table 3 (units=waves) and presented in Focus/Zemax format in Table 4 (units=inches). The surface 1 displacements after best-fit plane (BFP) and power have been removed (Figure 6) shows the distortion which cannot be corrected by pointing and focus adjustment.

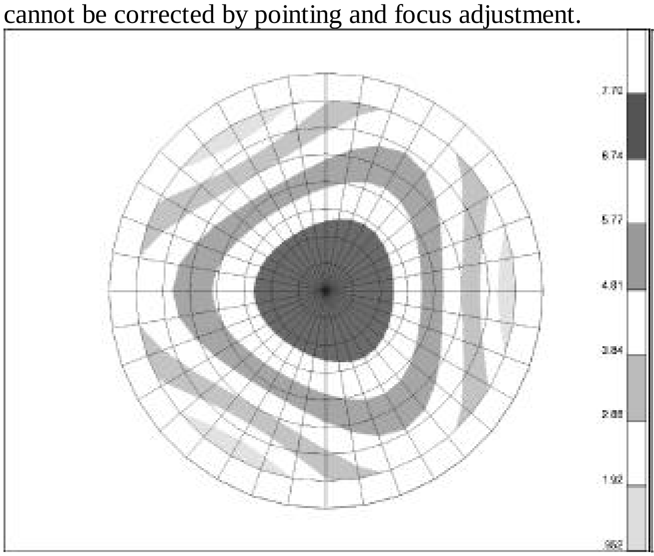

Figure 5 Corrected Displacements (RMS=5.1λ)

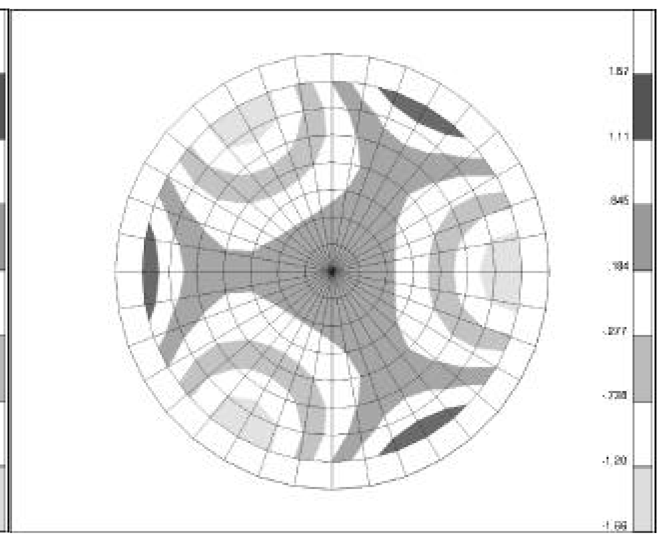

Figure 6 After BFP and Power removed (RMS=.66λ)

Table 3. Zernike fit to Surface 1 deformations

```
==============================================================================
Sigmadyne, Inc.       SigFit  Version=v2002-r2
------------------------------------------------------------------------------
    Surface  =  1     FE units  =      in          Wavelength= 2.4910E-05

    Order    Aberration         Magnitude   Phase     Residual    Residual
   K  N  M                       (Waves)    (Deg)       RMS          P-V
             Input(wrt zero)                           5.0850      6.7476
   1  0  0   Bias                 4.41187      .0      1.6724      6.7476
   2  1  1   Tilt                  .08074      .0      1.6719      6.8004
```

3	2	0	Power (Defocus)	-2.78048	.0	.6635	3.2274
4	2	2	Pri Astigmatism	.03661	-90.0	.6633	3.1900
5	3	1	Pri Coma	.03244	-180.0	.6632	3.1743
6	3	3	Pri Trefoil	2.11130	-60.0	.2194	.9435
7	4	0	Pri Spherical	.47800	.0	.0845	.3578
8	4	2	Sec Astigmatism	.00018	.0	.0845	.3578
9	4	4	Pri Tetrafoil	.01438	-45.0	.0844	.3499
10	5	1	Sec Coma	.02319	.0	.0842	.3396
11	5	3	Sec Trefoil	.28611	.0	.0397	.2344
12	5	5	Pri Pentafoil	.00296	.0	.0396	.2363
13	6	0	Sec Spherical	-.01797	.0	.0392	.2363
14	6	2	Ter Astigmatism	.00796	.0	.0391	.2329
15	6	4	Sec Tetrafoil	.00118	.0	.0391	.2329
16	6	6	Pri Hexafoil	.20787	-30.0	.0156	.0895

Table 4. Focus/Zemax representation

```
B$="SZERNSAG"
SURFTYPE   1,SCOD(B$)
EDVA   1,  1, 28
EDVA   1,  2,  2.50000E+00
EDVA   1,  3,  2.30527E-04
EDVA   1,  4,  1.00568E-06
EDVA   1,  5,  0.00000E-01
EDVA   1,  6, -3.99883E-05
EDVA   1,  7,  0.00000E-01
EDVA   1,  8, -3.72291E-07
…. truncated
```

How well the Zernike polynomials represent the surface can be determined by looking at the residual RMS after all terms are removed (RMS=.0156λ in Table 3) and comparing to the input RMS (.5.08λ) to find the percentage that is represented (99.7%). The residual is plotted in Figure 7. When the polynomials do not represent a good fit to surface distortions, an interferogram array may be written in the desired format. To calculate surface data on a rectangular array, 2D shape function interpolation[2] is applied to the FE arbitrary mesh. As a useful data check, SigFit provides a graphical representation of the interpolated data as seen in Figure 8. Interferograms are especially useful for high order deformations such as quilting, or for direct comparison to experimental interferogram test data. The analytically predicted interferogram can be used as a "backout" to correct 1g test data to a 0g environment.

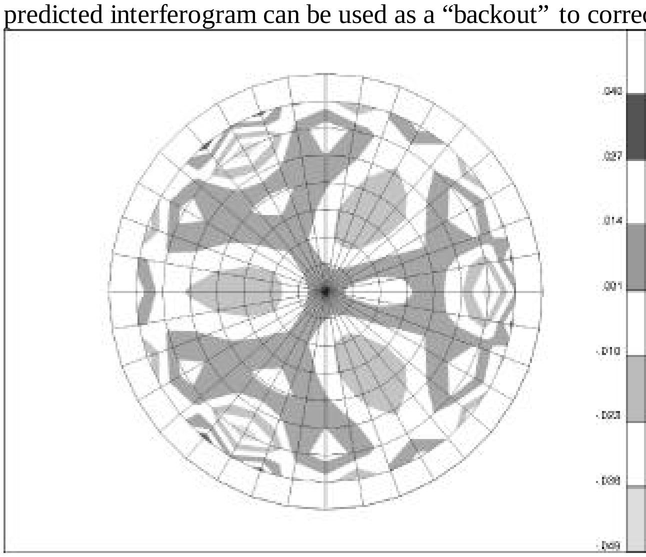

Figure 7. Residual after all terms removed

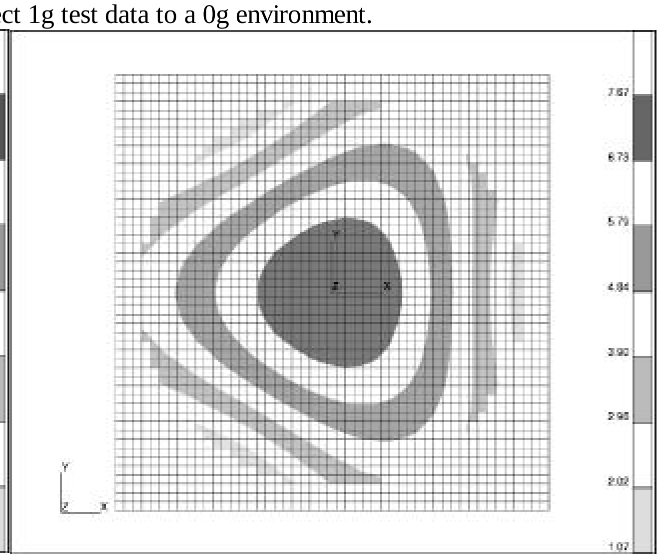

Figure 8. Interpolated interferogram array

4.2 dn/d s Effects

When subjected to stress (σ), the index of refraction (n) changes creating an optical path difference (OPD) and stress birefringence, called a dn/dσ effect . Consider a state of stress in which σ_{zz} is the normal stress along the optical ray and

σ_{11} and σ_{22} are the principal stresses in a plane normal to the ray. The index change in the orthogonal principal directions can be calculated from

$$\begin{aligned}\Delta n_1 &= k_{11} s_{11} + k_{12}\left(s_{22} + s_{zz}\right)\\ \Delta n_2 &= k_{11} s_{22} + k_{12}\left(s_{11} + s_{zz}\right)\end{aligned} \tag{4.1}$$

where k_{11} and k_{12} are the stress-optical coefficients of the material. The wavefront error (OPD) is the average index change effect whereas the polarization effect (ΔOPD) is represented by the difference in index changes.

$$\begin{aligned}OPD &= \frac{1}{2}\left(\Delta n_1 + \Delta n_2\right)L\\ \Delta OPD &= \left(\Delta n_1 - \Delta n_2\right)L\end{aligned} \tag{4.2}$$

Jones vectors are used to calculate rotation and retarder matrices from Δn1 and Δn_2 at each integration step and multiplied together to get a net effect, which is then converted to a birefringence (BIR) magnitude and crystal axis orientation (CAO)[9].

In SigFit, equations 4.2 are numerically integrated through the optic to create an OPD wavefront map and stress birefringence maps which may be output as a Zernike Polynomial table or an interferogram array in CodeV or Zemax format. A 3D solid finite element model of the optic and the element stresses are required as input. The steps within SigFit are

1) Read the model, element stresses, k_{11} and k_{12}, entrance (R_i) and exit (R_o) aperture radii
2) Convert stress to surface coordinate system and average at each node in the optic
3) From each node on the entrance surface, pass an average ray through the optic
4) For each integration step along the ray, use 3D interpolation[2] to find the state of stress
5) Transform the stress into the ray coordinate system and calculate principal stresses σ_{11}, σ_{22}
6) Numerically integrate and sum the OPD for each ray
7) Use Jones vectors to calculate rotation & retarder matrices, and convert to BIR and CAO
8) Fit Zernike polynomials[3] to the OPD,BIR, and CAO over the surface and write in optical format
9) Optionally, interpolate from FE mesh to regular interferogram array and output in optical format
10) Write graphical (nodal) files for viewing the OPD,BIR, and CAO results

For the lens in Figure 1 loaded with the temperatures in Figure 2, the stress OPD is shown in Figure 9 and the stress birefringence magnitude BIR is shown in Figure 10. This data can be passed to an optics program as Zernike or array data.

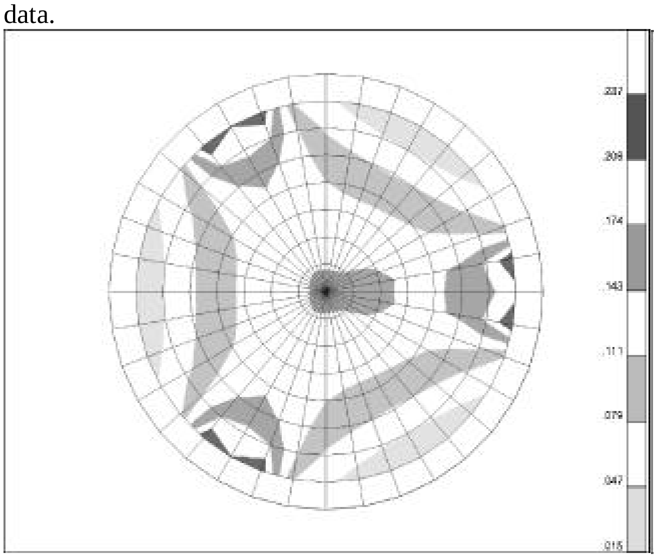

Figure 9 Stress OPD wavefront

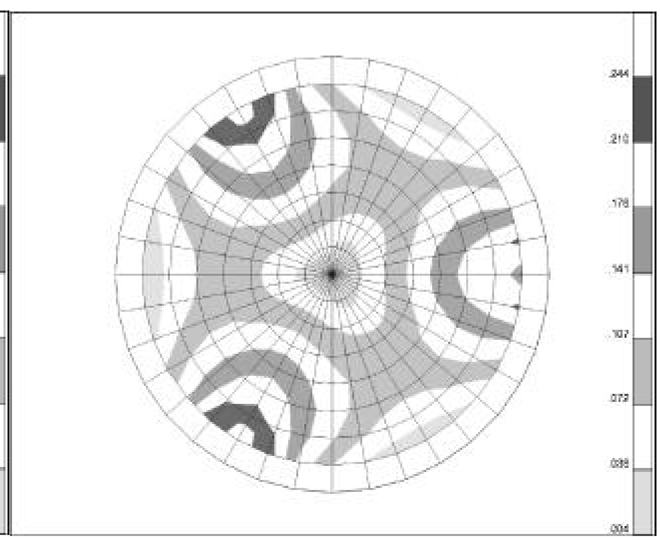

Figure 10. Stress Birefringence Magnitude

4.3 Vibrations

In vibrations analysis, each time step could be processed as the static load cases above. The large volume of data requires as much automation as possible. Even when surface distortion may not be significant, there are cases when the conversion of surface rigid body motion from FE to optical format is extremely useful. Consider an optical bench with several small optics in a variety of orientations subjected to dynamic loads. SigFit can convert the FE data at each desired time step into the proper optical format with appropriate coordinate transformations, numbering, and unit conversions.

When studying the behavior of an optical system, it is often useful to decompose the motion into rigid body motion which effects image motion and pointing, verses elastic distortion which impacts image quality. In SigFit, each mode is decomposed into rigid body motion and residual elastic motion[7]. Dynamic response (harmonic, transient or random) is calculated within SigFit to produce surface rigid body and surface RMS responses. In addition, the per cent contribution of each mode is presented for further understanding of the behavior and possible design improvements.

For a segmented primary mirror (Figure 11) mounted on a support structure (Figure 12) subjected to a random base excitation in the Y direction (Figure 13), SigFit calculates the random response of the surface RMS (Figure 14) with rigid body motion removed. The SigFit output (Table 5) shows the primary mirror random response decomposed into the 1σ rigid body motion and the 1σ surface RMS distortion. Table 6 shows the modal contributors to the 1σ results.

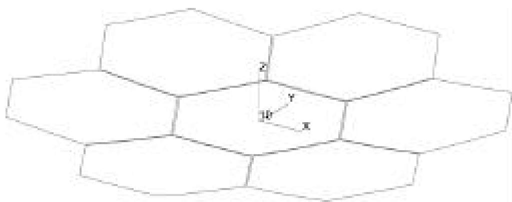

Figure 11. Primary Mirror with 7 Segments

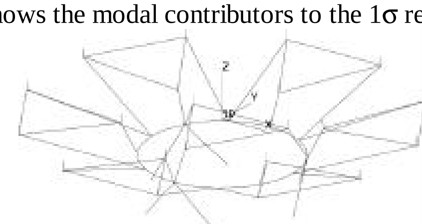

Figure 12. Primary Mirror Support Structure

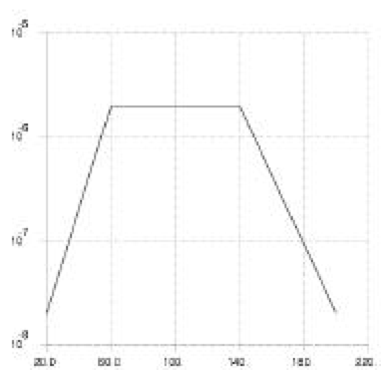

Figure 13. Base Input PSD

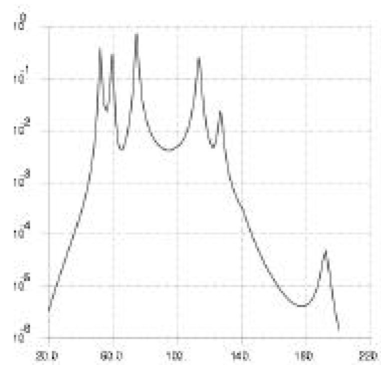

Figure 14. Surface RMS PSD

Table 5. PM Random Response

```
Rigid Body Translations (FE units): RB-Tx,...
Rigid Body Rotations (Radians): RB-Rx,...
S-RMS = Surface rms AFTER BFP removed (Waves)
                   <----------Displacement--------->
Surf#    Item       1-sigma      3-sigma      Zero-Xs
    1    RB-Tx     2.6156E-11   7.8468E-11   7.2854E+01
    1    RB-Ty     7.0861E-05   2.1258E-04   1.0046E+02
    1    RB-Tz     1.7986E-08   5.3957E-08   9.3310E+01
    1    RB-Rx     3.2146E-06   9.6437E-06   5.4058E+01
    1    RB-Ry     2.2890E-12   6.8671E-12   7.7506E+01
    1    RB-Rz     1.9179E-14   5.7536E-14   1.0925E+02
    1    S-RMS     2.0429E+00   6.1286E+00   8.1552E+01
```

Table 6. Modal Contributors to Random Response

Each modes % contribution to PSD for Surface= 1

Mode	Freq	RB-Tx	RB-Ty	RB-Tz	RB-Rx	RB-Ry	S-RMS
4	51.73	48.771	0.000	0.000	0.000	49.239	0.000
5	51.74	48.135	**15.758**	.820	96.622	49.691	**17.788**
6	57.88	0.000	0.000	1.516	0.000	0.000	0.000
7	59.46	.612	0.000	0.000	0.000	.523	0.000
8	59.46	.595	.210	6.566	1.093	.525	**13.670**
9	62.42	0.000	0.000	4.624	0.000	0.000	0.000
10	72.00	0.000	0.000	0.000	0.000	0.000	0.000
11	74.45	0.000	0.000	0.000	0.000	0.000	0.000

..... truncated

4.4 Adaptive Optics

Another structural-optical interface issue is the analysis of adaptive optics[7]. In many applications, such as large ground based astronomical telescopes, deformable mirrors are used to correct aberrations in the system wavefront which may be caused by deformations in the telescope or effects in the atmosphere. An FE model is used to calculate actuator influence functions as well as any structural distortions due to mechanical loading. SigFit can then be used to find the set of actuator scale factors which minimizes the surface error. The addition of augmented actuators allows this correction to be conducted before or after pointing and focus correction. The input surface and corrected surface are compared to calculate a correctability factor The input and corrected surfaces are fit to Zernikes, and optionally interpolated to interferogram arrays, for passing to optical codes. The input and corrected surfaces are also written to nodal files for graphical representation.

The 7 segment primary mirror (Figure 11 & 12) with 3 displacement actuators (rigid-body control) and 19 force actuators (distortion control) on each segment was subjected to a 1g lateral load and 55C uniform temperature increase. SigFit was used to calculate the behavior for various levels of actuator control. Figure 15 shows the deformed surface and Figure 16 removes the Best-Fit plane. In Figure 17, the effect of the displacement actuators on each segment is represented, and in Figure 18 the force actuators are used to correct the figure.

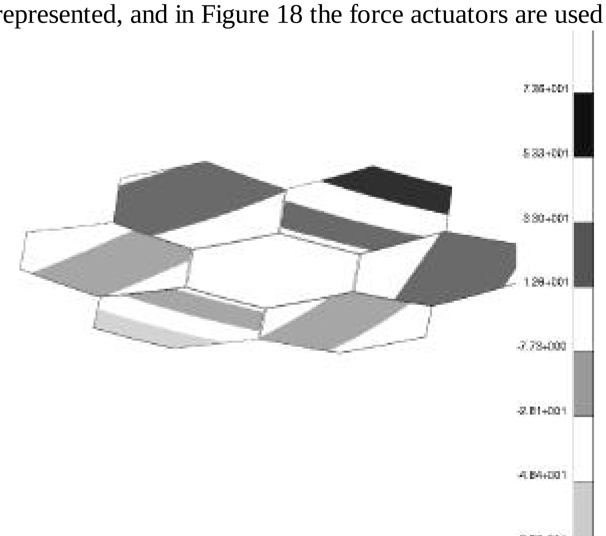

Figure 15. Deformed Surface

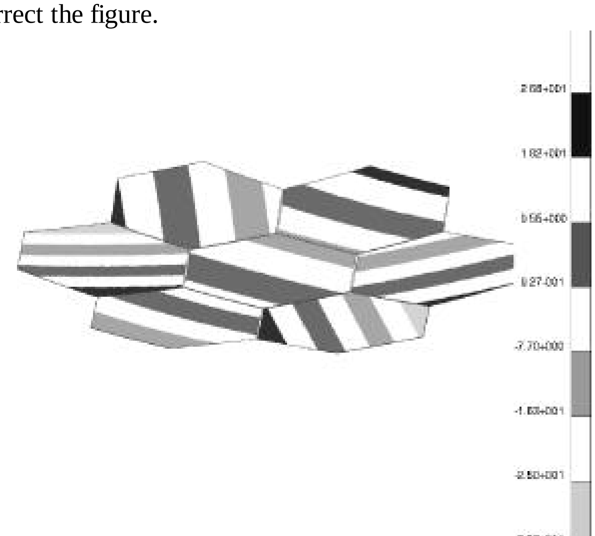

Figure 16. Best-Fit Plane removed

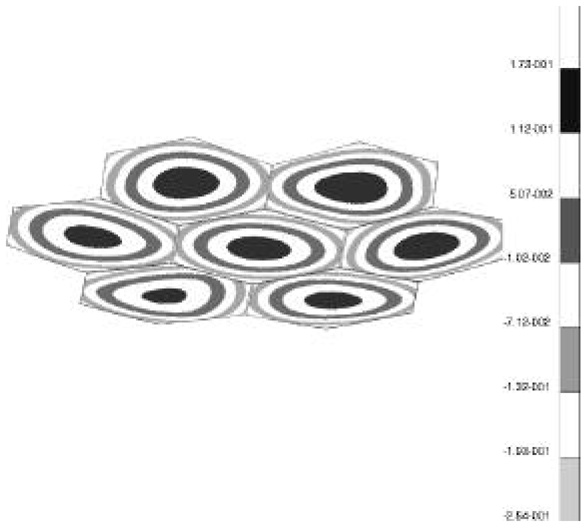

Figure 17. Petal Rigid Body Correction

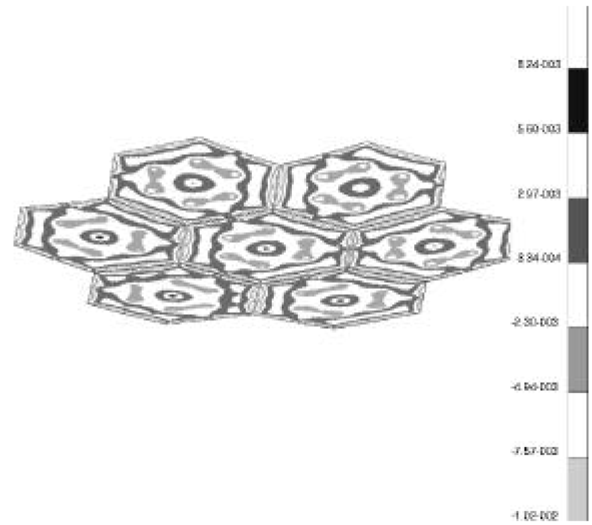

Figure 18. All Actuator Correction

5.0 SUMMARY

Standard finite element thermal and structural results are not in a useful form for the integrated performance analysis shown in Figure 1. This paper has presented a commercially available program (Sigmadyne/SigFit) which provides the necessary links to pass the thermal and structural data to popular optical analysis codes. Surface distortions can be passed as Zernike coefficient tables or interferogram arrays accounting for proper sign conventions, units, and coordinate transformations. The wavefront effects of dn/dT and dn/dσ, as well as stress birefringence effects can also passed from structural programs to optical programs. SigFit also calculates the behavior of adaptive optics and presents the results in optical code format. The interface tools discussed allows integrated performance analysis to be conducted effectively.

6.0 REFERENCES

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