Design optimization of actively controlled optics

Gregory J. Michels*, Victor L. Genberg
Sigmadyne, Inc., 803 West Avenue, Rochester, NY 14611

ABSTRACT

The finite element method is used to perform optimization of an actively controlled mirror's structural design. The theory of the method of modeling actuators is developed followed by execution of a test case demonstrating the effectiveness of this method in improving the correctability of a lightweight mirror. Design variables include shape and sizing optimization of the mirror's structural design. The design objective is the root-mean-square optical surface error after best correction of a wavefront with power aberration. Design constraints are applied to the mirror weight and the mounted natural frequency.

Keywords: Design optimization, active optics, finite element

1. INTRODUCTION

The current demand for active optics requires efficient methods by which efficient structural designs can be easily developed. To be effective such methods are required to relate optical performance quantities to structural design variables. The use of uncorrected optical performance quantities as responses in finite element optimization techniques has been developed in previous work.[1] Furthermore, post-processing methods are available for computing an actively controlled optic's wavefront correctability from finite element results of actuator influences. The method presented here, however, combines both of these capabilities to include actively corrected optical performance quantities as responses in structural finite element design optimization. By simultaneously including both actuator variables and structural design variables, an automated optimization analysis can tune the structural design to the actuator layout and also meet any other given requirements.

The method presented is demonstrated using MSC/NASTRAN v.70.7 but may be utilized in any finite element tool with similar design optimization capability.

2. THEORY OF METHOD

2.1. Optimization Problem Statement

The standard form of a design optimization problem is as follows:

MINIMIZE:	F
DESIGN VARIABLES:	$X_L \le X_i \le X_U$
SUBJECT TO:	$G_j = (R_j - R_U)/R_U \le 0$ $G_j = (R_L - R_j)/R_L \le 0$

where F is the objective function, X_i is the i^{th} design variable with lower and upper limits, X_L and X_U, respectively, and G_j is the j^{th} design constraint which is related to the j^{th} response quantity, R_j, whose allowable lower and upper bounds are R_L and R_U, respectively.

2.1.1. Design variables

Design variables are those quantities related to physical characteristics of the design problem that may vary within allowable ranges during the optimization process. For example, in the lightweight mirror test case to be presented in Section 4 the structural design variables include the faceplate thickness (T_p), core wall thickness (T_c), and various shape variables (H_k) for designing the mirror core sculpted shape. The design variables also include quantities related to the behavior of each actuator used in wavefront correction of the optic.

* Correspondence: Email: michels@sigmadyne.com; WWW: http://www.sigmadyne.com; Telephone: (716) 235-6892

2.1.2. Design responses

Primary design responses include any quantities computed by the finite element code without user definition. Such responses include nodal displacements, weight (W), and first natural frequency (f_n).

Synthetic design responses include any quantities computed via an equation involving primary design responses or other synthetic design responses. The root-mean-square (RMS) of an optical surface deformation is a response quantity which must be synthetically computed from nodal displacements via an equation. This quantity is treated as the objective function to be minimized. Methods of calculating this quantity within a finite element optimization after removal of any Zernike polynomial(s) is treated in Reference 1.

2.2. Method of Modeling Actuators

Modeling of actuators in a finite element model is shown schematically in Figure 1. Actuators are assumed to be stiff links providing a variable enforced relative motion between the optic and its mounting structure. Each actuator model consists of bar element pairs connected to the same two nodes to which a change in temperature is applied. As shown in Figure 1 the bars in each pair are given coefficients of thermal expansion, α, which are opposite in sign. The areas of each bar element pair, A_1 and A_2, are driven opposite directions by a single design variable referred to as an actuator control variable. Therefore, any change in the actuator control variable will cause the bar element pair to grow or shrink as the bar with increasing area dominates over its respective counterpart whose area decreases. The sum of the areas is held at a relatively large constant so that the sensitivity of the optical surface RMS deformation with respect to the actuator control variable is significant enough to yield a well posed optimization problem.

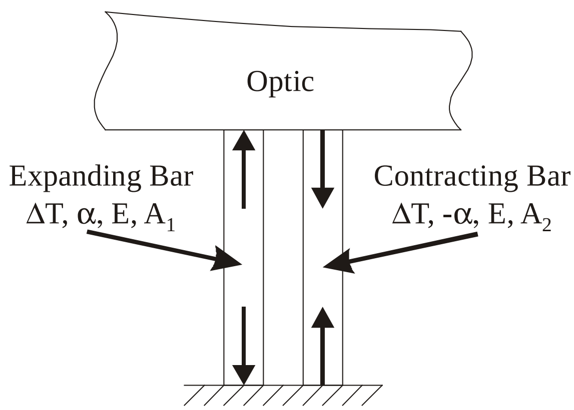

Figure 1: Schematic description of actuator model.

The relationship of the bar areas to the actuator control variable can be developed by starting with the assumption that the bar pair is very stiff compared to the rest of the system. Therefore, the only significant stiffness between the upper and lower ends of the bar pair is that of the two bar elements themselves. Therefore, we write the force balance at the top of the bar pair as

$$\frac{A_1 E}{L}(\alpha)(L)(\Delta T)+\frac{A_2 E}{L}(-\alpha)(L)(\Delta T)-\frac{A_1 E}{L}\delta-\frac{A_2 E}{L}\delta=0\,, \quad (1)$$

where A_1 and A_2 are the bar areas to be controlled by the actuator control variable, α is the coefficient of thermal expansion, L is the length, ΔT is an enforced change in temperature, E is the Young's modulus, and δ is the actuator displacement. Solving for the actuator displacement gives

$$\delta=\frac{-\frac{A_1 E}{L}(\alpha)(L)(\Delta T)+\frac{A_2 E}{L}(\alpha)(L)(\Delta T)}{-\frac{A_1 E}{L}-\frac{A_2 E}{L}}\,. \quad (2)$$

Canceling terms and setting L and ΔT arbitrarily to unity gives

$$\delta=\alpha\left(\frac{A_1-A_2}{A_1+A_2}\right). \quad (3)$$

We also require that the stiffness of the element pair remain constant as the areas are changed. Therefore,

$$A_1+A_2=A_T=\text{Constant}\,, \quad (4)$$

where A_T is a constant total area to be represented by the bar pair. Solving for A_2 in Equation (4) and substituting into Equation (3) yields,

$$\delta = \alpha\left(\frac{2A_1 - A_T}{A_T}\right), \tag{5}$$

which can be used to obtain an expression for A_1 in terms of the actuator displacement. An expression for A_2 can also be found in a similar manner. The final expressions are then

$$A_1 = \frac{A_T\delta}{2\alpha} + \frac{A_T}{2} \text{ and} \tag{6a}$$

$$A_2 = -\frac{A_T\delta}{2\alpha} + \frac{A_T}{2}. \tag{6b}$$

These relations prescribe that in order to increase δ, A_1 must be increased by the same amount A_2 is decreased. In order to present the optimizer with a well behaved design variable, the transformation,

$$\delta = \frac{(x_a - b)}{C}, \tag{7}$$

is made from δ to the actuator control variable, x_a, where b is an offset and C is a scaling factor. This transformation allows the actuator control variable, x_a, to range above and below a value b rather than zero thus avoiding numerical difficulties most optimizers have with zero valued design variables. It also allows control of the magnitude of the gradients of the objective function with respect to the actuator control variable through the scaling constant C.

Treatment of force actuators and displacement actuators is identical during any load case which computes the corrected surface as a design response. In other load cases or natural frequency computations in which it is desired that the force actuators be removed from the load path, a multi-point constraint, which is used to connect the force actuator bar pairs to the optic in the correctability load cases, can be turned off.

3. SPECIFIC GUIDELINES

Although very powerful tools in engineering and other disciplines, numerical optimizers are unfortunately not entirely robust. The usefulness of the final results are often highly dependent on the initial formulation of the design problem. These guidelines are listed as general recommendations specific to optimization of active optics in order to help the reader present the optimizer with a well posed design problem capable of returning useful results. However, these observations are not complete in advising the reader on the avoidance of ill-posed formulations. Further reading in optimization techniques may provide additional insight in coaxing more complicated design problems to useful solutions. [2,3]

3.1. Initial Actuator Inputs

The combined optimization process involving both actuator control variables and structural design variables proceeds more efficiently if the initial values of the actuator control variables are set to give the best corrected optical surface for the initial structural design. When these initial actuator inputs are present, the optimizer can more easily assess how to improve the corrected surface by adjustment of the structural design variables. The actuator control variables then only need to be given minor adjustments as the structural design variables evolve. The initial values for the actuator control variables can be found by performing an optimization on the initial structural design including only the actuator control variables.

3.2. Relative Scaling of Actuator and Structural Design Variable Sensitivities

When the optimizer is presented with initial actuator inputs as discussed in Section 3.1 it is very important that the first search direction have significant components in the structural design variables since consideration of only the actuator control variables represents an optimally corrected surface. Therefore, it is important that the structural design variable to property relations and the shape basis vectors be scaled such that the objective sensitivities with respect to the structural design variables are one or two orders of magnitude greater than the objective sensitivities with respect to the actuator control variables. Indication of failure to do this is evidenced by an optimization run that returns the initial structural design variable values after one design iteration. Examination of the objective gradient components will indicate by how much the structural design variable to property relations must be scaled in order to generate a well posed problem.

4. EXAMPLE PROBLEM

4.1. Design Optimization of an Actively Controlled Lightweight Mirror

Figure 2 shows the initial design of a 48 inch diameter actively controlled lightweight mirror fabricated of ULE which is to correct a wavefront with 4 He-Ne waves peak to valley of power error as defined by the $2\rho^2-1$ Zernike polynomial term. This amount of power corresponds to an optical surface error of 0.5793 He-Ne waves RMS. The mirror is to be controlled by 37 actuators whose locations are illustrated in Figure 3.

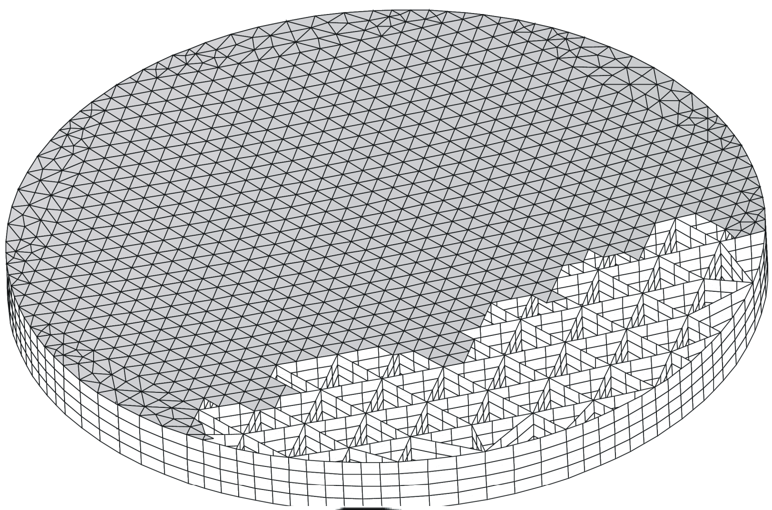

Figure 2: Finite element model plot of initial active mirror design. Top facesheet is partialy removed to show triangular core pattern.

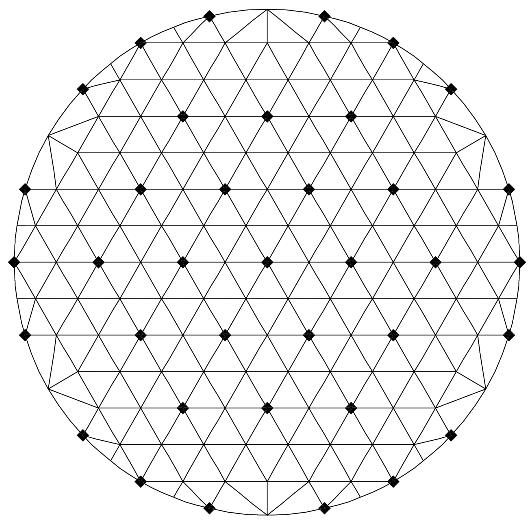

Figure 3: Locations of 37 actuators plotted on core layout.

The structural design variables are taken to be the faceplate thickness (T_p), core wall thickness (T_c), and five shape variables (H_k) for designing the mirror core sculpted shape. Each of the five shape basis vectors represents a different radially dependent core sculpting shape as illustrated in Figure 4.

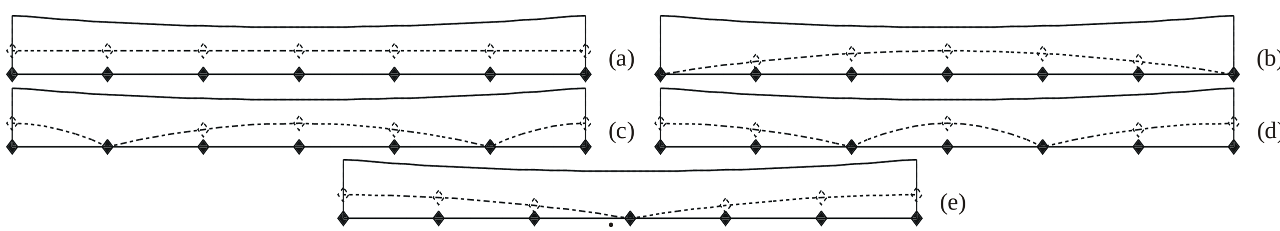

Figure 4: Plots of five shape variables. Solid lines show initial mirror profile while dashed lines show shape variation.

The encircled triangular cell size is held constant at 2.0 inches in diameter. It is required that the mirror weigh less than 70 lbs and that the first natural frequency on its three displacement actuators be greater than 150 Hz. The design optimization problem is written, therefore, as follows:

MINIMIZE:	Corrected Surface RMS
DESIGN VARIABLES:	$0.1 \text{ inch} \leq T_p \leq 0.5 \text{ inch}$ $0.04 \text{ inch} \leq T_c \leq 0.25 \text{ inch}$ $-1.0 \leq H_k \leq 1.0$ for k=1,2,3,4,5
SUBJECT TO:	$W \leq 70.0 \text{ lbs} \rightarrow (W-70.0)/70.0 \leq 0.0$ $f_n \geq 150.0 \text{ Hz} \rightarrow (150.0-f_n)/150.0 \leq 0.0$

An optimization with only the actuator control variables was performed initially to find the actuator inputs for the optimally corrected surface corresponding to the initial design. The resulting inputs were then used in a second optimization analysis which included both actuator control variables and structural design variables. The coefficient of thermal expansion for the actuator bar pairs was taken to be 0.01 $°F^{-1}$ while the Young's modulus and total pair area were set to 10^7 psi and 10.0 in^2, respectively. The actuator control variable transformation constants C and b were taken to be 10^4 and 1.0, respectively. Five finite element analyses were required to obtain a converged optimum. The performance results of the initial and optimized design are compared in Table 1.

Table 1
Performance Results of Active Mirror Optimization

Design Response	**Initial Design**	**Optimized Design**
Weight	93 lbs	70 lbs
Natural Frequency	188 Hz	150 Hz
Corrected Surface (0.5793 $\lambda_{He\text{-}Ne}$ RMS Input)	0.0152 $\lambda_{He\text{-}Ne}$ RMS	0.0089 $\lambda_{He\text{-}Ne}$ RMS

Even though the initial design violates the weight constraint, the resulting optimum design meets both the natural frequency and weight requirements with significant improvement in the corrected surface.

The optical faceplate thickness of the optimized design is 0.241 inch while the core wall thickness is 0.113 inch. Figure 5 shows the sculpted core profile of the optimum design.

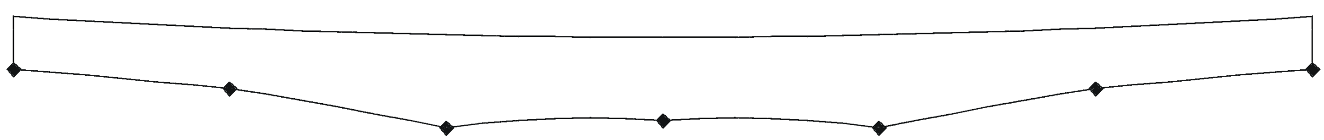

Figure 5: Plot of optimized mirror core shape.

5. CONCLUSIONS

A method for including the calculation of corrected surface of an actively controlled optic in structural design optimization is developed. Guidelines specific to this problem are offered to maximize the success of achieving useful optimization results. This method is also demonstrated by design optimization of an actively controlled lightweight mirror. The optimization of this example design results in a significant improvement in the corrected figure of the optical surface over the initial design. This improved correctability results from presenting the optimizer with the both structural design variables and actuator control variables in a single optimization analysis. With all of these variables the optimizer is able to tune the structural design variables to find an optimally correctable structure.

REFERENCES

1. V. Genberg, "Optical performance criteria in optimum structural design," *Proceedings of SPIE: Optomechanical Engineering and Vibration Control*, Volume 3786, pp. 248-255, 1999.
2. G. N. Vanderplaats, *Numerical Optimization Techniques for Engineering Design With Applications*, McGraw-Hill , New York, 1984.
3. C. Onwubiko, *Introduction to Engineering Design Optimization*, Prentice Hall, Upper Saddle River, New Jersey, 2000.