Optomechanical analysis of diffractive optical elements

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ABSTRACT

Diffractive optical elements are important components to many high precision optical systems. When such systems are subjected to mechanical loading these optical components yield performance degradation contributions quite different from non-diffractive optical components. It is of interest to predict by analysis such performance degradations for the purposes of development of the optomechanical design for relevant optical systems. The developments of this paper are to characterize the changes in phase due to such deformations as predicted by the finite element method and represent them in optical analysis alongside characterizations of surface shape changes.

Keywords: Diffractive optics, phase surfaces, integrated analysis, finite element analysis, optomechanics

1. INTRODUCTION

Figure 1 illustrates the flow of optomechanical analysis beginning with a thermal analysis and resulting in optical performance predictions. The topic of this paper focuses on characterizing surface deformations of diffractive optics for representation in optical analysis as indicated by the shaded path.

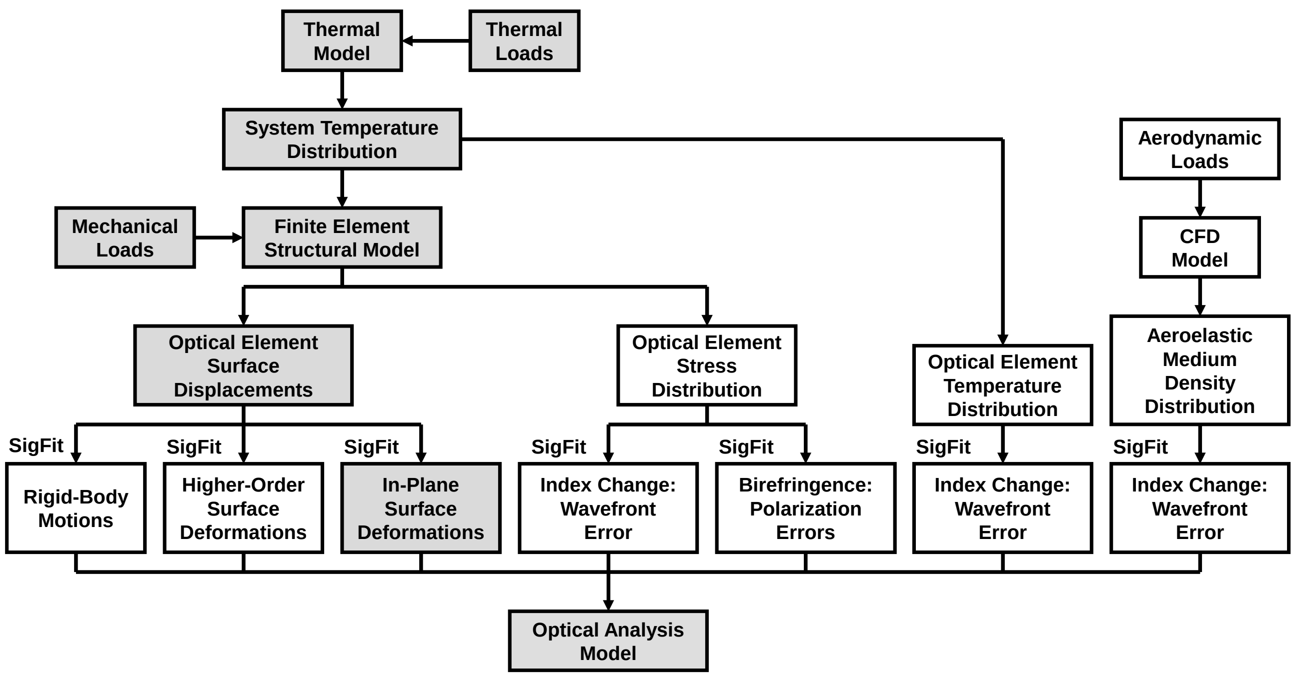

Figure 1: Flow chart of integrated optomechanical analysis with analysis of diffractive optics highlighted.

The characterization of the surface deformation of diffractive optics requires special treatment compared to that of conventional reflective and refractive surfaces as shown in Figure 2, in which the deformation of the undeformed dashed

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shape to the deformed solid shape is shown with darker gray vectors. Deformation of non-diffractive optical surfaces may be characterized by consideration of only sag surface deformation component as shown in Figure 2(a) with the lighter gray deformation vectors. This deformation results in a change in shape of the optical surface resulting in a deviation from the surface's nominal optical prescription. In the case of diffractive optics this substrate shape change is also important. However, deformations tangential to the surface, as shown in Figure 2(b) with the lighter gray vectors, cause changes to the phase properties of the diffractive optic. We will refer to these deformation components as the stretch deformations. These disturbances result in optical errors in addition to those caused by the substrate shape change disturbances shown in Figure 2(a).

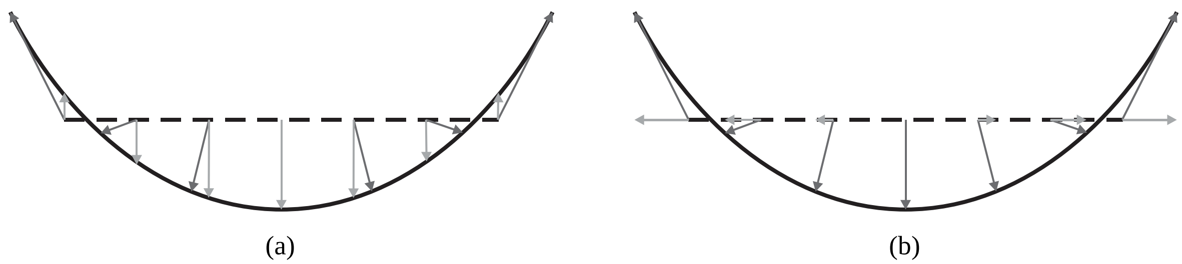


Figure 2: Surface deformation of non-diffractive and diffractive optical surfaces.

2. PHASE CHANGE CHARACTERIZATION OF DIFFRACTIVE OPTICS

2.1 Calculation of Phase Change of Linear Gratings

Figure 3(a) and (b) shows an example deformation of a planar linear grating. The deformation in the plane of the substrate results in changes to the spacing size of the rulings resulting in phase errors in the wavefront leaving the grating.

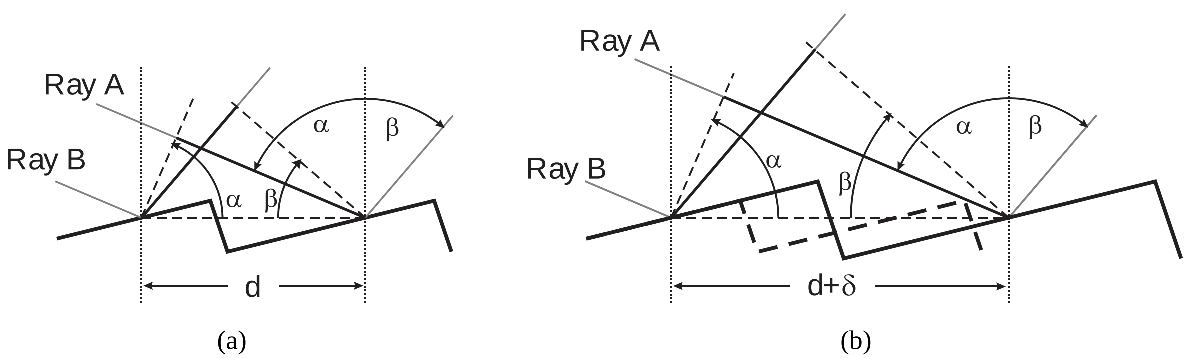


Figure 3: Illustration of phase change due to spacing change of a linear grating: (a) nominal grating and (b) deformed grating.

For Figure 3(a) we can express the phase difference between Ray A and Ray B [1] as,

$$\Phi_{AB} = -d\left(\sin\alpha - \sin\beta\right) \tag{1}$$

where, d is the grating spacing, α is shown as a positive angle, and β is shown as a negative angle. If we choose ray angles, α and β, that are integer multiples of the wavelength, λ, then we get,

$$-d\left(\sin\alpha - \sin\beta\right) = m\lambda\,, \tag{2}$$

where, m is the diffractive order. If the grating then deforms with a constant strain $\varepsilon = \delta/d$ then we may write the change in phase given by Eqn. (1) as follows:

$$\Delta\Phi_{AB} = -\left(d+\delta\right)\left(\sin\alpha - \sin\beta\right) + d\left(\sin\alpha - \sin\beta\right) = -\delta\left(\sin\alpha - \sin\beta\right) \tag{3}$$

We can now substitute from Eqn. (2).

$$\Delta\Phi_{AB} = -\frac{\delta}{d} m\lambda \tag{4}$$

In general the deformation is a displacement vector and must be decomposed into the deformation normal to the cutting planes of the grating. We may then write an expression for the change in phase relative to an arbitrarily chosen point as shown in Eqn. (5).

$$\Delta\Phi(x,y) = -\frac{\vec{\delta}(x,y)\cdot\vec{p}}{d} m\lambda \tag{5}$$

where, $\Delta\Phi(x,y)$ is the change in phase expressed in length units, $\vec{\delta}(x,y)$ is the displacement vector at the location (x,y) relative to an arbitrary reference point, $\vec{p}$ is the unit normal vector to grating cut planes, d is the constant grating spacing between cutting planes, m is the diffractive order, and λ is the use wavelength. Note that the choice of the reference point is arbitrary because it results in only a bias to the wavefront.

2.2 Calculation of Phase Change of Phase Surfaces

For phase surfaces the deformation induced phase can be found by a relation similar to that shown in Eqn. (5). However, in a phase surface the spacing size, d, must be represented by a spatially varying effective spacing size $d_{eff}(x,y)$. To define the effective spacing size a construction wavelength, λ_c, is introduced as shown in Eqn. (6). [2]

$$d_{eff}\left(x,y\right) = \frac{\lambda_c}{\sqrt{\vec{\nabla}\Phi(x,y)\cdot\vec{\nabla}\Phi(x,y)}} \tag{6}$$

where, $\Phi(x,y)$ is the nominal phase of the phase surface. Similarly, the vector, $\vec{p}$, is replaced by a spatially varying vector field, $\vec{a}(x,y)$, given by the unit vector in the direction of the gradient of the nominal phase.

$$\vec{a}\left(x,y\right) = \frac{\vec{\nabla}\Phi(x,y)}{\sqrt{\vec{\nabla}\Phi(x,y)\cdot\vec{\nabla}\Phi(x,y)}} \tag{7}$$

The change in phase may then be written as,

$$\Delta\Phi(x,y) = -\frac{\vec{\delta}(x,y)\cdot\vec{a}(x,y)}{d_{eff}(x,y)} m\lambda = -\frac{\vec{\delta}(x,y)\cdot\vec{\nabla}\Phi(x,y) m\lambda}{\lambda_c}. \tag{8}$$

2.3 Representation of Disturbed Diffractive Surfaces in Optical Analysis

In general, the change in phase associated with the deformation of a diffractive surface may vary over the surface. This yields phase changes that require polynomial fitting or array interpolation for representation in optical analysis. Additionally, the change in phase, like the nominal phase, is a wavelength dependent quantity and, therefore, must be represented in optical analysis as a wavelength dependent phase surface.

In general, the nominal phase and change in phase must be represented in a single characterization. Therefore, polynomial fitting or array interpolation is performed on the total phase of the disturbed diffractive surface, which is the combination of the nominal phase and the change in phase. In order to maintain the best accuracy in representation of the total disturbed phase the polynomial type used to define the nominal phase of a phase surface should be used to characterize the change in phase. In the case of characterization of the phase of deformed linear gratings the polynomial type selected must be able to represent the linear variation in phase associated with the nominal phase of a linear grating.

3. EXAMPLES

3.1 Example of Phase Change in a Linear Grating Due to Free Isothermal Growth

An example linear grating shown in Figure 4 has a flat round optical surface with a diameter of 10.0 mm and a CTE of 13.0 ppm/C. Constraints are applied in a kinematic manner at three support locations.

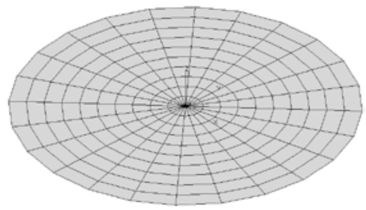


Figure 4: Finite element model of flat round diffractive optical surface.

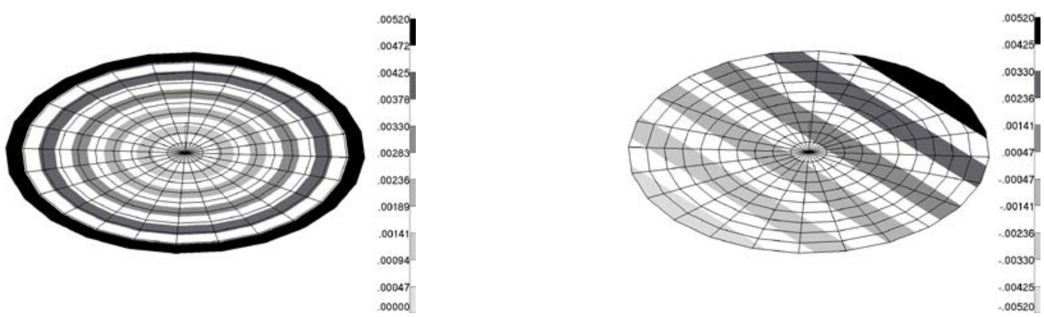
The linear grating applied to the surface of the optic has a spacing size of 2.0 μm with the rules oriented along the *x* direction of the vertex coordinate system. The nominal phase may be expressed in waves as,

$$\Phi(y) = \frac{y}{d} = \frac{y}{0.002} = 500.0y\,. \tag{9}$$

where, *y* is expressed in millimeters. An isothermal temperature change of 80.0C is considered causing a uniform change in spacing associated with stress free thermal growth. The displacement function may be written analytically in millimeters as,

$$\vec{\delta}(x,y) = \alpha\,\Delta T\;x\,\hat{i} + \alpha\,\Delta T\;y\;\hat{j} = 0.00104x\,\hat{i} + 0.00104y\;\hat{j}\,, \tag{10}$$

where, *x* and *y* are locations in the vertex coordinate system expressed in millimeters, $\hat{i}$ is the unit vector in the *x* direction, and $\hat{j}$ is the unit vector in the *y* direction. Figure 5 shows contour plots of displacement in the radial direction and in the direction perpendicular to the grating rules due to the isothermal free growth loading condition.



(a) (b)

Figure 5: Plots of displacement expressed in millimeters (a) in the radial direction and (b) in the direction perpendicular to the grating rules due to the isothermal free growth loading condition.

From Eqn. (5) the change in phase may then be written as,

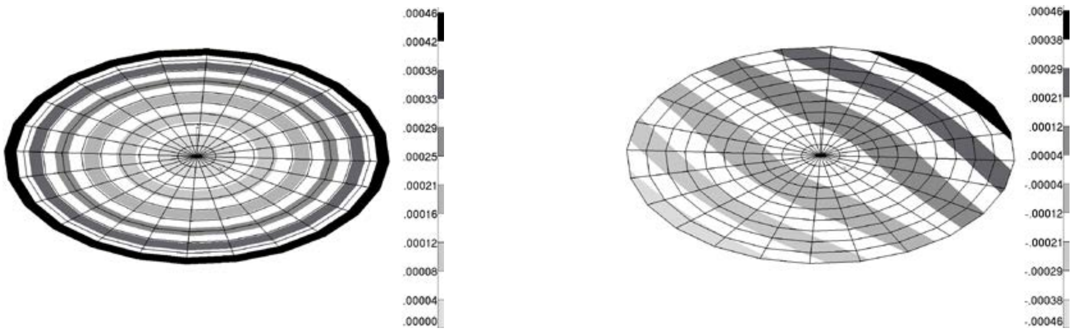
$$\Delta\Phi(y) = -\frac{(0.00104x\,\hat{i} + 0.00104y\;\hat{j})\cdot\hat{j}}{0.002}\,m\lambda = -0.52\,ym\lambda \tag{11}$$

where, *y* and is expressed in millimeters. For *m* = 1 this gives a change in phase of -0.52*y* waves.

In a second load case an axisymmetric cubic radial deformation is imposed. The enforced displacement vector is written in micrometers as,

$$\bar{\delta}(x, y) = 0.0416x(x^2 + y^2)\,\hat{i} + 0.0416y(x^2 + y^2)\,\hat{j}\,. \tag{12}$$

Figure 6 shows contour plots of displacement in the radial direction and in the direction perpendicular to the grating rules due to the isothermal free growth loading condition.



(a) (b)

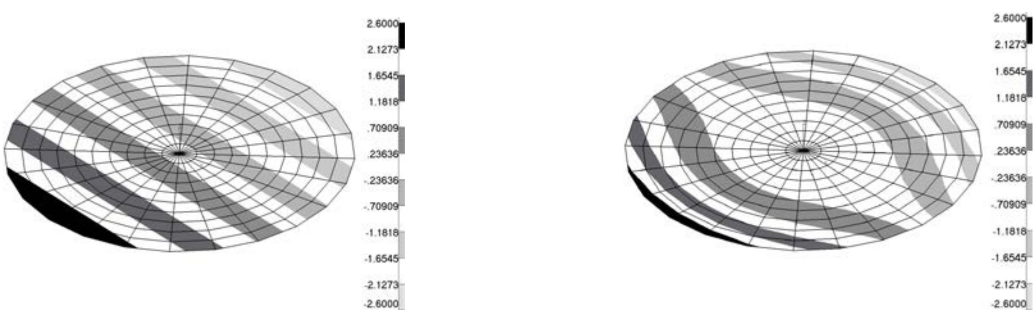
Figure 6: Plots of displacement expressed in millimeters (a) in the radial direction and (b) in the direction perpendicular to the grating rules.

From Eqn. (5) the change in phase may be written as,

$$\Delta\Phi(x, y) = -\frac{(0.0416x(x^2 + y^2)\,\hat{i} + 0.0416y(x^2 + y^2)\,\hat{j}) \cdot \hat{j}}{2.0} m\lambda = -0.0208y(x^2 + y^2)m\lambda\,, \tag{13}$$

where, x and y are expressed in millimeters. For m = 1 this gives a change in phase of -0.0208x^2y - 0.0208y^3 waves.

The changes in phase of the linear grating associated with the deformations computed by the finite element model were then computed in SigFit to obtain the change in phases as shown in Figure 7. [3]



(a) (b)

Figure 7: Plots of the change in phase in the linear grating expressed in waves for (a) the isothermal temperature change with free growth and (b) the radial cubic enforced deformation.

Polynomial fits of the total disturbed phase were fit in SigFit with XY polynomials. The resulting coefficients are shown in Table 1 compared to those found above by analytical means.

Table 1: Coefficients of Polynomial Fits of Change in Phase Due to Deformations in Linear Grating

	Isothermal ΔT		Cubic Radial Deformation	
Polynomial Term	**Analytic Solution (waves)**	**Numerical Solution (waves)**	**Analytic Solution (waves)**	**Numerical Solution (waves)**
y	-0.52	-0.52	0.0	0.0
x^2y	0.0	0.0	-0.0208	-0.0208
y^3	0.0	0.0	-0.0208	-0.0208

The characterizations of the phase disturbances were also exported to formats importable to popular optical analysis tools for subsequent optical analysis.

3.2 Example of Phase Change in a Phase Surface

The example model shown in Figure 4 is now considered as a phase surface having a nominal phase function of $100r^2$ waves where r is the radial location in millimeters. This phase function can equivalently be defined in micrometers with a construction wavelength of 0.55 μm as,

$$\Phi(x,y) = 100\lambda_c r^2 = 100(0.55)(x^2+y^2) = 55.0(x^2+y^2) \text{ for } \lambda_c\text{=0.55μm,} \tag{14}$$

where, x and y are expressed in millimeters. To compute the change in phase due to a deformation of the substrate Eqn. (8) may be employed. The gradient of the nominal phase may be written in micrometers as,

$$\vec{\nabla}\Phi(x,y) = 110.0x\,\hat{i} + 110.0y\,\hat{j} \text{ for } \lambda_c\text{=0.55μm.} \tag{15}$$

For the isothermal free growth deformation case the associated deformation vector given in Eqn. (10) and the phase gradient vector given in Eqn. (15) can be used to compute the change in phase as,

$$\Delta\Phi(x,y) = -\frac{(0.00104x\,\hat{i} + 0.00104y\,\hat{j})\cdot(110.0x\,\hat{i} + 110.0y\,\hat{j})\mathrm{m}\lambda}{\lambda_c} = -\frac{(0.1144x^2 + 0.1144y^2)\mathrm{m}\lambda}{\lambda_c}. \tag{16}$$

For the enforced deformation case the associated deformation vector given in Eqn. (12) and the phase gradient vector given in Eqn. (15) can be used to compute the change in phase as,

$$\begin{aligned}\Delta\Phi(x,y) &= -\frac{(0.0416x(x^2+y^2)\,\hat{i} + 0.0416y(x^2+y^2)\,\hat{j})\cdot(110.0x\,\hat{i} + 110.0y\,\hat{j})\mathrm{m}\lambda}{\lambda_c} \\ &= -\frac{(4.576x^2(x^2+y^2) + 4.576y^2(x^2+y^2))\mathrm{m}\lambda}{\lambda_c} \\ &= -\frac{(4.576x^4 + 9.152x^2y^2 + 4.576y^4)\mathrm{m}\lambda}{\lambda_c}\end{aligned} \tag{17}$$

For m = 1, λ_c = 0.55 μm, and this gives a change in phase of $-4.576x^4 - 9.152x^2y^2 - 4.576y^4$. Polynomial fits of the total disturbed phase were fit in SigFit with XY polynomials. The resulting coefficients are shown in Table 2 compared to those found above by analytical means.

Table 2: Coefficients of Polynomial Fits of Change in Phase Due to Deformations in Phase Surface

	Isothermal ΔT		Radial Gradient	
Polynomial Term	**Analytic Solution (μm)**	**Numerical Solution (μm)**	**Analytic Solution (μm)**	**Numerical Solution (μm)**
x^2	-114.4	-114.4	0.0	0.0
y^2	-114.4	-114.4	0.0	0.0
x^4	0.0	0.0	-4.576	-4.576
x^2y^2	0.0	0.0	-9.152	-9.152
y^4	0.0	0.0	-4.576	-4.576

The characterizations of the phase disturbances were also exported to formats importable to popular optical analysis tools for subsequent optical analysis.

4. SUMMARY

A method for computing the change in phase of mechanically disturbed diffractive optics was developed and demonstrated. The demonstration was performed analytically and numerically through the use of a finite element model and a new feature in the SigFit software. Characterization of the change in phase was performed using polynomial fitting methods compatible with import into optical analysis tools.

REFERENCES

[1] Diffraction Grating Handbook, Newport Corporation, 2005.

[2] Klein, James E., “Demystifying the Sequential Ray Tracing Algorithm,” Proc. SPIE 4769, Optical Design and Analysis Software II, 67, 2002.

[3] SigFit is a trademark of Sigmadyne, Inc.