# Novel applications of active mirror analysis

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#### Abstract

Active and adaptive mirrors are commonly used to improve the performance of telescopes and other high performance optical systems. This paper will address the use of this analysis capability to solve a variety of other optomechanical problems that are not related to active mirrors.

#### Introduction

Active and adaptive mirrors are commonly used to improve the performance of telescopes and other high performance optical systems<sup>1</sup>. In active mirror analysis, an input disturbance is corrected by actuators. The input disturbance may be in the incoming wavefront or it may be due to distortions of the optics in the system. Since incoming wavefronts can be high order, the correction is usually made by a small deformable mirror (DM) with a dense array of actuators (bed of nails). Thermoelastic and gravity distortions of large primary mirrors are generally low order and can be corrected by actuators on the primary mirror. Many variations of active mirrors exist in practice and three are shown in Figure 1. The analysis mathematics, which is based on linear superposition, can be used to solve a variety of other optomechanical problems.



Figure 1: Active mirror configurations

## Background

In this paper, the surface error (or wavefront) to be corrected is called a disturbance. The surface distortion created by a unit value of actuator stroke is called in influence function. After correction, the remaining uncorrected surface error is the residual surface, usually measured as RMS.



Figure 2: Actuator correction

The residual error (E) is written as the disturbance minus the linear combination of actuator influence functions.

$$E = \sum_{i=1}^{N} w_{i} \left( ds'_{i} - \sum_{j=1}^{M} A_{j} f_{ji} \right)^{2}$$

N = the number of nodes

M = the number of actuators

 $ds'_i$  = input disturbance = surface deformation at node i

 $A_j$  = actuator stroke (input) for actuator influence function j

 $f_{ji}$  = surface deformation of actuator influence function *j* at node *i* 

 $w_i$  = surface area fraction associated with node i

Taking partial derivatives of E with respect to each actuator input  $A_i$  and setting each equal to zero leads to a linear system of M equations with M unknowns

$$[H]{A} = {F}$$
$$H_{jk} = \sum_{i} w_{i} f_{ji} f_{ki}$$
$$F_{k} = \sum_{i} w_{i} ds'_{i} f_{ki}$$

SigFit<sup>2</sup> has an option to include slope error ( $\Theta$ ,  $\Phi$ ) in addition to displacement which is important for X-ray optics<sup>4</sup>. In the following equation, slope fraction factor (c) is user specified.

$$E = (1-c)\sum_{i=1}^{N} w_i \left( ds'_i - \sum_{j=1}^{M} A_j f_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Theta'_i - \sum_{j=1}^{M} A_j \Theta_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{N} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} A_j \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left( d\Phi'_i - \sum_{j=1}^{M} W_i \Phi_{ji} \right)^2 + (c)(L)\sum_{i=1}^{M} w_i \left($$

Actuators may have fixed stroke limits (lower limit=*L* and upper limit=*U*)

$$\{L\} \leq \{A\} \leq \{U\}$$

For thin deformable mirrors, relative stroke limits are required to prevent mirror damage (Figure 3).



8" diameter mirror; 49 actuators; 1" spacing

#### Figure 3: Deformable mirror

In SigFit, relative stroke limits are written as linear inequalities.

$$\alpha_1 A_1 + \alpha_2 A_2 + \ldots + \alpha_n A_n \le C$$

For dense arrays, SigFit has a feature to create the many constraints automatically. When constraints are added to the system, SigFit uses a nonlinear programming algorithm to solve for actuator strokes.

## **Special Features**

SigFit<sup>3</sup> has several features for active analysis. Since actuators have limits on their stroke resolution, a tolerance analysis using Monte Carlo techniques is useful to predict 'real' performance verses perfect theoretical performance. Figure 4 shows surface correction with perfect actuators.



**Figure 4: Active correction with perfect actuators** 

Using 'perfect' actuators, corrected surface RMS is  $0.052 \lambda$ . If actuators had a 1% variablity, the corrected surface RMS is  $0.079 \lambda$  for 95% confidence.

A common design problem is deciding how many actuators are required and where should they be placed. SigFit uses genetic optimization<sup>6</sup> to solve this problem. Figure 5 shows all possible actuator locations on a mirror. The 18 best are shown which provide the minimum corrected surface RMS for specified load or multiple load conditions.



Figure 5: Actuator placement optimization

Using this feature, design curves (Figure 6) can be created for surface correction verses the number of actuators. Note that these are not any set on N actuators, but a 'best' set of N actuators.



Figure 6: Corrected surface verses number of actuators.

In the telescope model shown in Figure 7, the adaptive primary mirror was structurally optimized to satisfy system level wavefront error (WFE) under thermal and gravity loads<sup>7</sup>. The design variables were core and face thickness of the mirror as shown in Figure 7. Design constraints were placed on mirror stress, natural frequency, and system level WFE. The WFE of the actuator corrected telescope was calculated from a linear optics model within SigFit. The optimization was conducted in MSC/Nastran's solution 200 which called SigFit as a DRESP3 subroutine to calculate the adaptive mirror correction and the system WFE. The resulting design met all performance requirements while cutting the mirror weight in half (Figure 8).



Figure 7: Telescope and optimized adaptive primary mirror

Response	Initial Design	Optimized Design	Requirement
Thermally Induced Wavefront Error	9 nm	20 nm	20 nm
Gravity Release Induced Wavefront Error	54 nm	60 nm	60 nm
Peak Launch Stresses	1000 psi	1000 psi	1000 psi
First Natural Frequency	231 Hz	221 Hz	200 Hz
Weight	20.8 kg	9.9 kg	Minimum
Areal Density	53.0 kg/m <sup>2</sup>	25.2 kg/m <sup>2</sup>	Minimum

Figure 8: Optimization results

# **Key feature**

The key feature which allows novel applications is the variety of input options for disturbances and actuator influence functions as shown in Figure 9.



Figure 9: Input options in active analysis

Test data, analysis results, polynomials, and external data may be linearly combined for disturbance or actuators. Some examples will be given in the next section.

## **Novel applications**

## Example 1:

Metal mirrors require a coating to get the required surface reflectivity. This coating cause a bi-metallic effect which will distort the mirror under temperature changes. For uniform thickness mirrors, the design solution is to coat the back surface with the same coating thickness. For light-weight mirror designs, the solution is not so obvious. In Figure 10, the designer has the option to coat various portions of the back of the mirror. Treating this as an active analysis problem, the disturbance is the distortion caused by the front surface coating under a given temperature change. The actuator influence functions are the distortions caused by individually coating each back section separately. Active analysis will then solve for the best coating thickness in each region to minimize surface RMS in a single solution. No time consuming trade study of various thickness combinations is required.



Figure 10: Back surface coating example

			limit			
	All 4	1,3,4	1,3,4	limit 3,4		
	Thick	Thick	Thick	Thick		
Coating	(mil)	(mil)	(mil)	(mil)		
1	1.49	7.12	5.00			
2	4.58					
3	2.85	2.73	2.96	3.77		
4	2.92	2.72	0.00	0.00		
%corr	99.6	96.4	95.2	81.5		
with Tol	96.6	93.7	92.4	78.9		
Correctability for isothermal temperature change						
Tolerance of 0.1 mil on thickness control						
Limits on coating thickness 0.0 < thick < 5.0						

Figure 10B: Optimized thickness and correctability

Once the 4 unit actuator cases are created, several variations are available within SigFit. To understand the effect of not coating an area, just ignore that subcase. In Figure 10B, the first column is with all 4 zones coated at their optimized thickness with a correctability of 99.6%. If zone 2 was not coated, the new thicknesses would have 96.4% correctability. Suppose a zone 1 thickness of 7.12 mil was undesirable, the analysis can be rerun with a 5 mil limit in column 3. Without a lower bound of 0.0, a negative thickness (non-physical) for zone 4 was obtained. If the coating group insisted that only zone 3 be coated, correctability dropped to 81.5%. The correctability predictions assume accurate thickness control. Using SigFit's Monte Carlo tolerancing<sup>8</sup> capability, the effect of coating control limited to 0.1 mil is shown in the last row of the table. This would represent a more realistic correctability prediction.

#### Example 2:

A mirror with three mount pads bonded to the back face was being tested on two air bag rings. The air bag pressures were adjusted to minimize surface RMS after focus. The resulting gravity sag was desired so that it could be subtracted from the test results, but the pressures were unknown.



Figure 11: Air bag test support

Active analysis was used to solve for the pressures. The disturbance was the gravity sag. The actuator influence functions were surface distortions under pressure P1 and pressure P2. Bias and Tilt were added to the actuator set to account for quantities that could not be measured by the optical test. An additional influence function of Zernike power was added to account for the subtraction of focus from the test results.

#### Example 3:

A mirror mounted on an internal hub (Figure 12) was tested in gravity with a two orientation test. The test results showed that 0.15 waves of astigmatism were present in the mounted mirror. In searching for the cause many possibilities were eliminated. The cause was finally determined through active analysis to be mount plate flatness. The disturbance was the locked in astigmatism. The actuator influence functions were unit normal displacements at the 6 bolt locations on a mount plate (Figure 13).



Figure 12: Mounted mirror distortion



Figure 13: Active solution

## Example 4:

The surface figure of a large mirror mounted on three whiffle trees changed after a random vibration test. To understand possible causes, active analysis was applied in SigFit. The disturbance was a linear combination of two test interferogram files, post-test minus pre-test, interpolated on to the FE model.

The interferogram files were directly read into SigFit and subtracted, then interpolated on the the FE mirror model. The actuator influence functions were unit differential mount rotations in two directions simulating

local flexure yield at each of 9 mount locations. The 18 influence functions were determined from FE analysis. Active analysis determined how much differential slip at each mount was required to best match the test data.



**Figure 14: Explaining test anomolies** 

# **Other examples :**

Active analysis can be used to solve other design problems and to understand perplexing test results.

- 1) The thermoelastic effect of coating thickness variation
- 2) The curing effect of adhesive bond thickness variation
- 3) Mirror distortions caused by mount effects such as flexure bending or misalignment
- 4) Material property variation such as CTE variation through a mirror substrate
- 5) Stressed-optic polishing<sup>5</sup> and stressed-lap polishing

## Summary

Active mirror analysis may be used to solve a variety of optomechanical problems. The feature may be used to find the best linear combination of actuator influence functions to minimize or match a disturbance function

In SigFit, actuator influences AND disturbances may be any of:

- Finite element subcase solutions <u>anything</u> FE can solve (not just local forces)
- Test data in the form of interferogram array files
- Any set of polynomials including rigid body motion,  $\Delta RoC$ , Zernike, XY, etc
- Any externally created vector array of data from any source
- Any linear combination of the above

Limits may be applied on actuator stroke, both fixed limits or relative limits through inequalities. The effect of tolerances can be studied using Monte Carlo simulation. The number of actuators is only limited by computer resources.

#### **References (download from Sigmadyne.com)**

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