

Structural Optimization of System WFE

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ABSTRACT

The ultimate design goal of an imaging optical system subjected to thermal and dynamic loads is to minimize system level wavefront error (WFE). System WFE is impossible to predict from finite element random response results due to the loss of phase information. In the past, the use of system WFE was limited by the difficulty of obtaining a linear optics model (LOM). In this paper, an automated method for determining system level WFE using a linear optics model is presented. The technique is applied to a simple telescope using structural optimization to automatically handle the conflicting design requirements of thermal and random response loads. The technique is demonstrated by example with SigFit, a commercially available tool integrating mechanical analysis with optical analysis.

Keywords: STOP analysis, Wave Front Error, LOS jitter, Optimization, Finite Elements, Random Response

1. INTRODUCTION

Increasing performance requirements in high precision astronomical instruments have created the need for increased capability in predicting their performance when subjected to operational environments. Both ground based and space based environments contain thermal variations and vibration disturbances that must be considered in the design development of such high precision systems. Figure 1 shows a flow chart of integrated analysis.^[1]

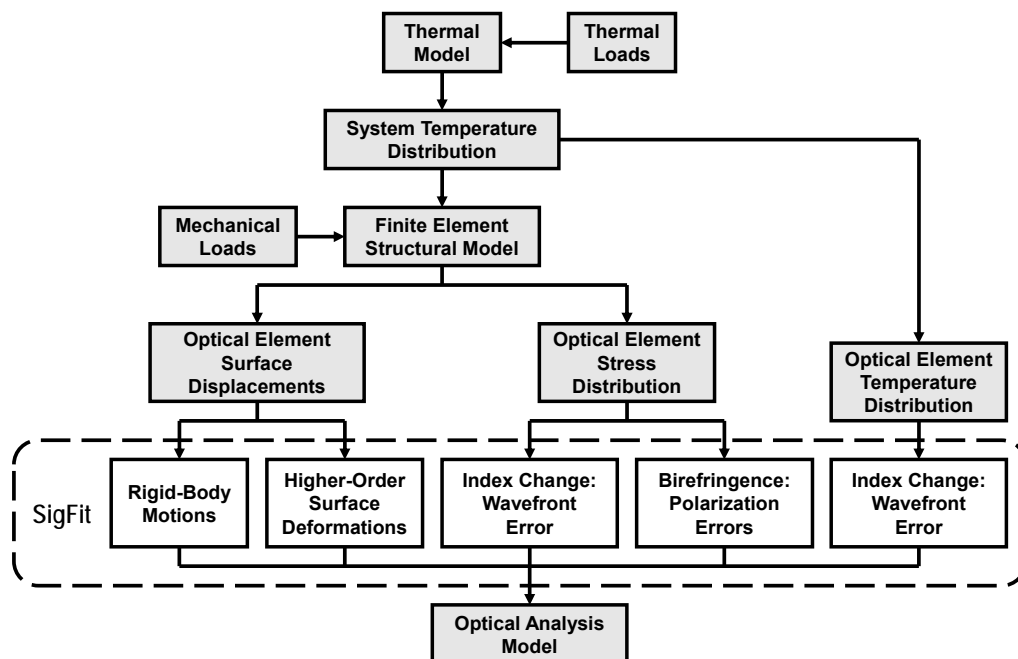


Figure 1. Flow chart of integrated STOP analysis.

As shown in Figure 1, system WFE degradation is primarily caused by three physical phenomena: surface deformation and refractive index changes due to stress or temperature in transmissive optics. SigFit^[2] is commercial

software that interfaces to finite element (FE) programs such as Nastran™, Ansys™, Abaqus™, and SolidWorks Simulation™ and passes data to optical design codes such as Code V™, Zemax Optic Studio™, and Oslo™ to predict optomechanical effects.

In Figure 1, the static deformation of each surface for each load case is fit with polynomials and passed to optics code for evaluation. This paper addresses the complex issues of passing finite element (FE) results of dynamics analyses to optics codes. The issues include the following:

- 1) In transient response, the response of each surface at each time step must be fit with polynomials.
- 2) In harmonic analysis, the response is complex because of damping. Each FE node has its own phase angle. At any frequency step, the response must be fit at each phase angle (0 to 360) and fit with polynomials to determine the peak optical response.
- 3) In random analysis, only the magnitude of each node's displacement is known. All phasing information between nodes is lost. In Figure 2. If three nodes all had equal magnitude, then it is impossible to tell the difference between (a) or (b).



Figure 2: Random response output (a) all rigid body (b) all elastic

2. LINEAR OPTICS MODEL FOR PREDICTION OF SYSTEM WAVEFRONT ERROR

2.1 Linear Optics Model

As an alternative to the approach shown in Figure 1 a method that brings knowledge of the optical model into the FEA analysis may be employed. This is done through a linear optics model (LOM), which is obtained using the following approach in SigFit™ [2]

- 1) Create an optical analysis model of the system using a commercial optical design code
- 2) Within SigFit, read the optics model and choose the type and order of polynomials on each surface to represent surface error. Typically Zernike polynomials are used.
- 3) SigFit then conducts the following steps automatically
 - a. Place surface error described by a rigid body motion (RBM) of finite difference magnitude on the first surface
 - b. Command the optical code to calculate the polynomials of WFE over the pupil
 - c. Repeat steps a and b for all RBMs and polynomials of surface error on the first surface
 - d. Repeat steps a, b, c for all surfaces in the model
- 4) SigFit collects all of the polynomials of WFE, normalizes them by their respective finite difference steps and assembles them into a matrix [S]
- 5) SigFit then reads FE disturbances and fits RBMs and polynomials of surface error to the optical surfaces [C]
- 6) System WFE polynomials [Z] are computed from equation 1
- 7) System WFE RMS is calculated from the RSS of [Z] where w converts magnitude to RMS

- N = Total number of optical surfaces in model
- T_n = Total number of RBMs and polynomial terms of surface error fit to surface n
- i = Static Load case
- S_{kt}^n = LOM sensitivity matrix (generated as described above)
= Zernike polynomials coefficient of WFE k at pupil for RBM and polynomial of surface error t at surface n
- C_{it}^n = RBM or polynomial coefficient of surface error t characterizing FE displacements of load case i and surface n
- Z_{ki}^0 = Zernike polynomial of WFE k at pupil for load case i
- w_k = Conversion factor to convert Zernike polynomial k to RMS

$$Z_{ki}^0 = \sum_n^N \sum_t^{T_n} S_{kt}^n C_{it}^n \quad (1)$$

Since the Zernike polynomials of WFE are orthogonal over the pupil, the WFE RMS can be found from the root-sum-squared (RSS) of the polynomial coefficients of WFE where w is the conversion from polynomial magnitude to polynomial RMS.

$$WFE_RMS = RSS(w_k Z_k^0) \quad (2)$$

In step 3 above, the optics code is called as many times as required to create the full LOM matrix [S]. The matrix [S] may be saved to a file for subsequent analyses. Until the optical prescription is changed, step 3 is replaced with a call to read the [S] matrix from file, which improves solution efficiency.

The major improvement in the above approach is that the calculation of the LOM matrices is totally automated in SigFit. Since the LOM is created through SigFit, the polynomial ordering, amplitude normalization and radial normalization are all consistent with SigFit's rigid body motion and polynomial characterization of the surface error disturbances.

2.2 System WFE Prediction in Dynamic Analysis

The previous section's steps can be applied to vibration analysis by including the rigid body motion and surface error polynomial fits to the dynamic mode shapes.

- N = Total number of optical surfaces in model
- T_n = Total number of RBMs and polynomial terms of surface error fit to surface n
- M = Total number of dynamic modes used
- i = Frequency step or time step
- S_{kt}^n = LOM sensitivity matrix (generated as described above)
= Zernike polynomials coefficient of WFE k at pupil for RBM and polynomial of surface error t at surface n
- C_{mt}^n = RBM or polynomial coefficient of surface error t characterizing FE displacements of mode m and surface n
- z_{mi} = Modal response function (mode factor) of mode m at frequency or time step i (complex in harmonic response analysis)
- Z_{ki}^0 = Zernike polynomials of WFE k at pupil for frequency or time step i
- w_k = Conversion factor to convert Zernike polynomial k to RMS

The dynamic response of WFE can then be computed as any other response quantity as shown in equation (3) by scaling by the modal response function, z_{mi} , of mode m at each time step i or frequency step i determined using standard modal analysis techniques.

$$Z_{ki}^0 = \sum_n^N \sum_t^{T_n} \sum_m^M S_{kt}^n C_{tm}^n z_{mi} \quad (3)$$

For computational efficiency, each mode shape may be fit once and scaled by the LOM, $[S]$, to get an eigen-WFE vector for each mode, which can be assembled into a matrix $[A]$ as shown in equation (4) and utilized as shown in equation (5).

$$A_{km}^0 = \sum_n^N \sum_t^T S_{kt}^n C_{tm}^n \quad (4)$$

$$Z_{ki}^0 = \sum_m^M A_{km}^0 z_{mi} \quad (5)$$

$$WFE_RMS = RSS(w_k Z_k^0) \quad (6)$$

The accuracy of this approach depends on the quality of the polynomial fit of the mode shapes of the optical surfaces. SigFit reports fitting errors from which the user can judge overall accuracy of the method.

2.3 Flow Chart for System WFE in Dynamics Analysis

Figure 3 shows the analysis flow to get system WFE response in dynamic analysis using a LOM. Through a COM API to the optical analysis software SigFit can generate the LOM consistent with the characterization of the FE modal results. This facilitates a seamless LOM generation for the user. The process flow also allows the user to optionally run the forced response analysis in the FEA software or in SigFit. If the forced response analysis is performed in the FEA software then the modal response functions are used by SigFit in combination with the eigenvector results from the modal analysis. If forced response analysis is run in SigFit then the user must supply the dynamic loads, damping, and either frequency steps for harmonic and random analysis or time steps for transient analysis. Problems for which the damping or loading is complex it is easier to run the forced response analysis in the FEA and allow SigFit to use the modal response functions.

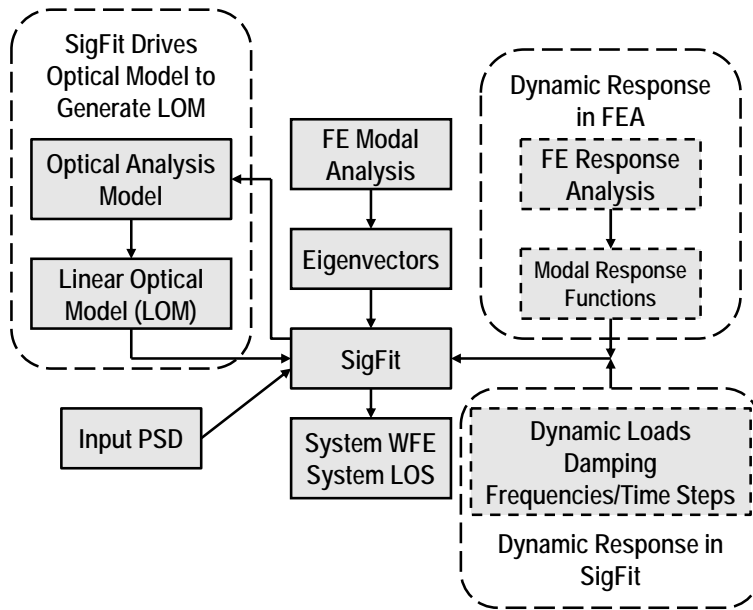


Figure 3: System WFE analysis in dynamic analysis

3. OPTIMIZATION USING SYSTEM WAVEFRONT ERROR

3.1 Structural Optimization of WFE in FEA

The flow chart in Figure 4 shows the two step process to use system WFE as a constraint or objective function in FE optimization optimization solution. The left block shows SigFit equation generation to obtain equations of WFE and LOS as inputs to the FEM. These equations allow the FEA to compute WFE and LOS during FE design optimization. The right block shows the FE design optimization flow. The SigFit equation generation analysis needs to be rerun only if the optical design changes or the finite element model of the optical surfaces change. The remainder of the telescope may be changed during the design optimization process.

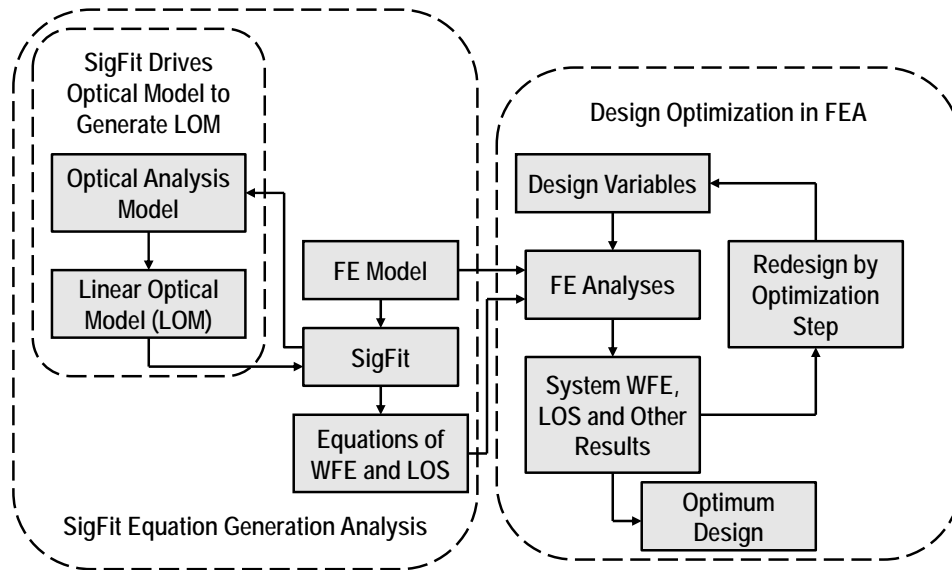


Figure 4: Optimization flow

With this feature, system WFE is a design response that may be constrained or minimized. SigFit writes a variety of optically important terms as bulk data equations [3].

- System WFE polynomial terms written as MPC equation for Nastran
- System net WFE RMS written as DRESP2 for Nastran
- System LOS written as MPC for Nastran and ANSYS
- Surface Rigid-Body motion (with radial correction) written as MPC equation for Nastran and ANSYS
- Surface displacements with RB subtracted written as DRESP1 & MPC for Nastran
- Surface Zernike/other polynomial terms written as MPC equation for Nastran
- Surface residual RMS written as DRESP2 equation for Nastran

4. EXAMPLE TELESCOPE OPTIMIZATION

4.1 Simple Telescope

The simple telescope shown in Figure 5 is used to demonstrate the optimization technique using the flow chart in Figure 4. In Figure 5 the naming used is PM for primary mirror, SM for secondary mirror, and FP for focal plane.

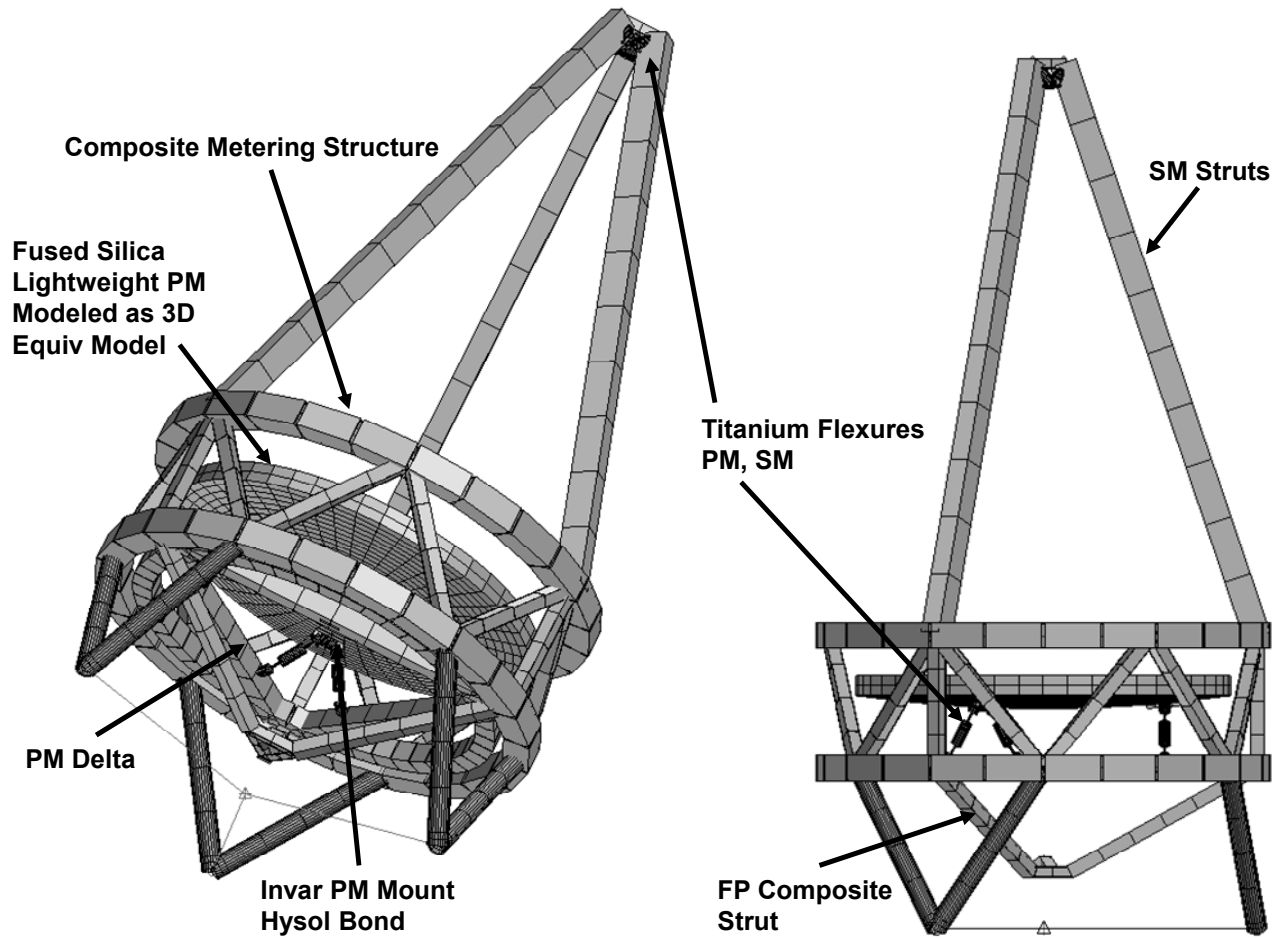


Figure 5: Simple telescope FE model

The design variables include:

- Primary Mirror (PM) front face, back face, and core thickness
- PM and SM flexure size
- SM struts and PM delta frame size

The loads considered are:

- Isothermal temperature change
- Random base shake

The constraints used are:

- System WFE in thermal < 0.07 waves after focus correction
- System WFE in random forced response < 0.07 waves at 3-sigma and no focus correction
- Line-of-sight (LoS) in random forced response < 25 microns at 3-sigma

Design objective

- Minimize total weight

Other load conditions (such as launch or test setup) and other constraints (such as stress, buckling, etc) can be included for a more realistic example. The purpose here is to show that including system WFE as a design constraint does affect the design.

4.2 Optimization Results

Table 1 gives the design variable results and Table 2 shows the response results.

| Design Variable | | Start | Final | Low Limit | Up Limit |
|-----------------|---------|-------|-------|-----------|----------|
| PM-FLEXURE | (inch) | 0.075 | 0.100 | 0.050 | 0.100 |
| PM-DELTA | (inch) | 1.500 | 2.500 | 0.050 | 2.500 |
| SM-FLEXURE | (inch) | 0.050 | 0.040 | 0.040 | 0.100 |
| SM-STRUT | (inch) | 1.500 | 1.023 | 1.000 | 2.500 |
| FP-STRUT | (inch) | 1.000 | 0.641 | 0.500 | 2.500 |
| PM-FRONT | (inch) | 0.040 | 0.031 | 0.030 | 0.060 |
| PM-BACK | (inch) | 0.040 | 0.060 | 0.030 | 0.060 |
| Core Density | (ratio) | 0.015 | 0.011 | 0.010 | 0.030 |

Table 1: Design variables

| Response | | Start | Final | limit |
|-------------|----------|-------|-------|-------|
| Thermal WFE | (waves) | 0.075 | 0.070 | 0.07 |
| Random WFE | (waves) | 0.065 | 0.061 | 0.07 |
| Random LOS | (micron) | 59.3 | 25 | 25 |
| Weight | (Lbs) | 57.1 | 55.3 | |

Table 2: Design responses

The initial design violated the system WFE requirement. The optimizer increased the PM back plate thickness to reduce PM deformation, which has the most significant impact on the WFE. The initial design also violated the LoS requirement for the random base vibration case. In this case the optimizer increased the PM flexure and the PM delta frame to reduce the PM motion. The optimizer also reduced the PM front faceplate and core wall thicknesses to reduce the PM weight and thus reduce random response. Other design variables are reduced to drive down the objective of minimum overall weight. This example shows that system WFE can be used in a design optimization to obtain effective design improvement results.

5. MANUAL DESIGN TRADES

The approach described in section 4 applies to FE optimization capability. However, manual design trades can use most of the capabilities for system WFE and LOS.^{[4],[5],[6]} Figure 3 above can be used with any FE code that interfaces to SigFit. Not only will SigFit calculate the LOS and WFE response, but will also identify which modes are the key contributors^{[8],[9],[10]}. The analyst can now plot those mode's strain energy density to assist in design changes to improve response.

When SigFit calls the optical analysis to create the LOM matrices, the matrices are written to an ASCII file for future SigFit runs. The LOM only needs to be recreated when the optical design changes. The LOM file is an efficiency feature to make subsequent SigFit runs faster and allow analysis to be run by users of SigFit who may not have access to a license of the optical analysis software. Since the LOM is an ASCII file, the analyst can delete selected matrices to understand which component's rigid body motion or elastic distortion is important to the system WFE.

6. EXTENSIONS

The linear optics model capability has been extended to all SigFit surface deformation analysis types.

- 1) Fitting: system WFE polynomials and plots of WFE created

- 2) Active: corrected system WFE polynomials and plots of WFE created ^[4]
- 3) Harmonic Response: system WFE transfer functions are added to output (peaks noted)
- 4) Transient Response: system WFE time history added to output files (peaks noted)
- 5) Random Response: system WFE PSD functions, 1-sigma response, and modal contributors added to output
- 6) State Space equation output: system WFE include in response equations
- 7) Monte Carlo analysis: tolerance of system WFE to variations added to output ^[7]
- 8) Equation writing: system WFE equations written to finite element model (for optimization)

7. CONCLUSIONS

The automated linear optics model approach described above will allow design decisions to be based on the system response rather than individual component responses. This paper shows that system WFE can be used in a design optimization to affect the final design.

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