

# Using integrated models to minimize environmentally induced wavefront error in optomechanical design and analysis

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## ABSTRACT

The ultimate design goal of an optical system subjected to dynamic loads is to minimize system level wavefront error (WFE). In random response analysis, system WFE is difficult to predict from finite element results due to the loss of phase information. In the past, the use of system WFE was limited by the difficulty of obtaining a linear optics model. In this paper, an automated method for determining system level WFE using a linear optics model is presented. An error estimate is included in the analysis output based on fitting errors of mode shapes. The technique is demonstrated by example with SigFit, a commercially available tool integrating mechanical analysis with optical analysis.

**Keywords:** Finite Element, Random Response, Vibrations, WaveFront Error, System Analysis, Optomechanical

## 1. INTRODUCTION

Increasing performance requirements in high precision astronomical instruments have created the need for increased capability in predicting their performance when subjected to operational environments. Both ground based and space based environments contain thermal variations and vibration disturbances that must be considered in the design development of such high precision systems. Figure 1 shows a flow chart of integrated analysis.<sup>[1]</sup>

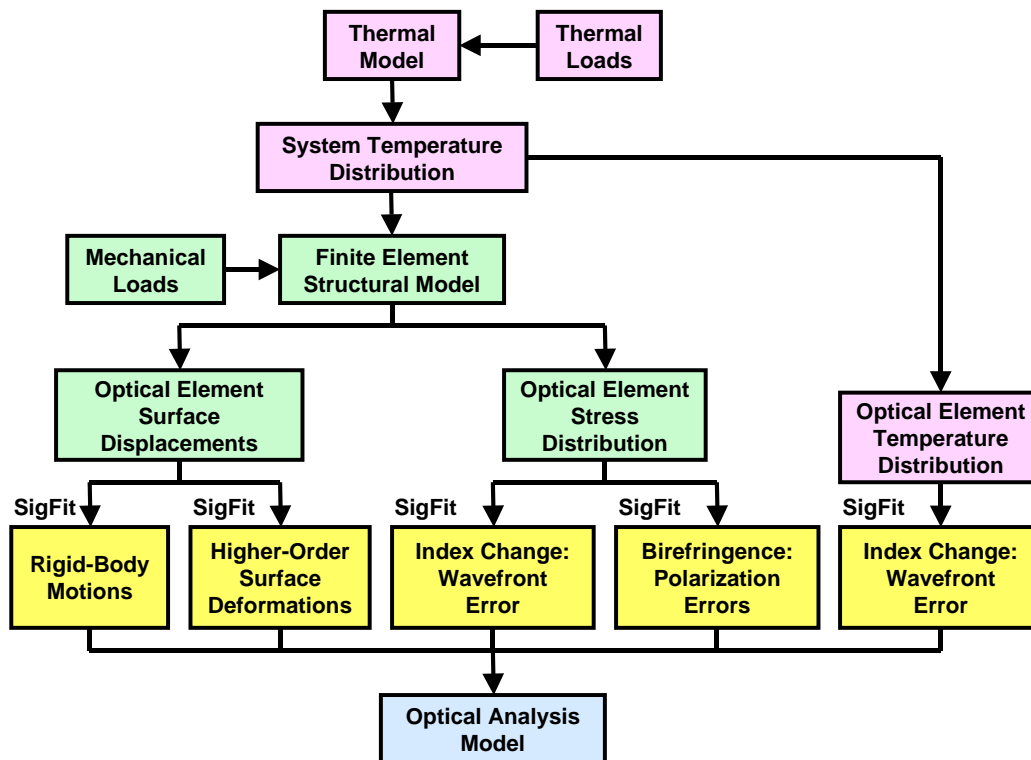
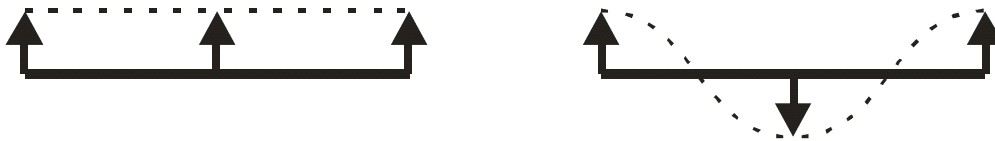


Figure 1. Flow chart of integrated optomechanical analysis.

As shown in Figure 1 disturbances are primarily caused by three physical phenomena: surface deformation, refractive index changes due to stresses in transmissive optics, and refractive index changes due to temperature changes in transmissive optics. SigFit<sup>[2]</sup> is commercial software that interfaces to finite element (FE) programs such as Nastran, Ansys, Abaqus, and SolidWorks Simulation and passes data to optical design codes such as CodeV, Zemax, and Oslo to predict optomechanical effects.

In Figure 1, the deformation of each surface in each load case is fit with polynomials and passed to optics code for evaluation. The issues with passing finite element (FE) results to optics codes in dynamic analysis are the following:

- 1) In transient, the response of each surface at each time step must be fit with polynomials.
- 2) In harmonic analysis, the response is complex because of damping. Each FE node has it's own phase angle. At any frequency step, the response must be fit at each phase angle (0 to 360) and fit with polynomials to determine the peak optical response.
- 3) In random analysis, only the magnitude of each node's displacement is known. All phasing information is lost. In Figure 2. If three nodes all had equal magnitude, then it is impossible to tell the difference between (a) or (b).



**Figure 2: Random response output (a) all rigid body (b) all elastic**

## 2. LINEAR OPTICS MODEL FOR PREDICTION OF SYSTEM WAVEFRONT ERROR

### 2.1 Linear Optics Model

The linear optics model (LOM) is obtained from the following approach in SigFit <sup>[2]</sup>

- 1) Create an optical analysis model of the system using a commercial optical design code
- 2) Within SigFit, read the optics model and choose the type and order of polynomials on each surface
- 3) SigFit then conducts the following steps automatically
  - a. Place a polynomial on surface one
  - b. Call the optical code to calculate polynomials at the focal plane
  - c. Repeat steps a and b for all polynomials on surface 1
  - d. Now repeat steps a, b, c for all surfaces in the model
- 4) SigFit collects all of the polynomials at the focal plane into a large matrix [S]
- 5) SigFit now reads FE disturbances and fits polynomials to the optical surfaces [C]
- 6) System WFE polynomials [Z] are computed from equation 1
- 7) System WFE RMS is calculated from the RSS of [Z] where w converts magnitude to RMS

N = Total number of optical surfaces in model

T = Total number of Rigid-Body (R-B) and Zernike terms fit to each surface

i = Static Load case

$S_{kt}^n$  = Sensitivity matrix (from optical code)

= Zernike (k) at exit pupil for each R-B / Zernike (t) at surface (n)

$C_{it}^n$  = Static displacement decomposition of FE displacements

= fit each deformation case (i) with R-B /Zernike (t) at surface (n)

$Z_{ki}^0$  = Zernikes (k) at output location (0) for load step (i)

=  $Z^0$  is the system level response at desired location (exit pupil)

$$Z_{ki}^0 = \sum_n^N \sum_t^T S_{kt}^n C_{it}^n$$

$$WFE\_RMS = RSS(w_k Z_k^0)$$

In step 3 above, the optics code is called several times to create the full LOM matrix [S]. The matrix [S] is saved to a file for subsequent analyses. Until the optical prescription is changed, step 3 is replaced with a call to read the [S] matrix from file, which improves solution efficiency.

The major improvement in the above approach is that the calculation of the LOM matrices is totally automated in SigFit. Since the LOM is created within SigFit, the polynomial ordering, amplitude normalization and radial normalization are all consistent with SigFit's disturbance fitting polynomials. All SigFit features are under user control for selection of polynomials.

## 2.2 System WFE prediction in dynamic analysis

The previous section steps can be applied to vibration analysis by including the polynomial fit to the dynamic mode shapes.

N = Total number of optical surfaces in model

T = Total number of Rigid-Body (R-B) and Zernike terms fit to each surface

M = Total number of dynamic modes used

i = Load step number (frequency step, or time step)

$S_{kt}^n$  = LOM matrix = Zernike (k) at exit pupil for each R-B / Zernike (t) at surface (n)

$C_{tm}^n$  = Mode (m) decomposition into R-B /Zernike (t) at surface (n)

$z_{mi}$  = modal partition (mode factor) of mode (m) at step I (complex in harmonic)

$Z_{ki}^0$  = Zernikes (k) at output location (0) for load step (i)

$W_k$  = weighting factor to convert Zernike (k) magnitude to WFE RMS

$$Z_{ki}^0 = \sum_n^N \sum_t^T \sum_m^M S_{kt}^n C_{tm}^n z_{mi}$$

For computational efficiency, fit each mode shape once and multiply by [S] to get A= WFE for each mode.

$$A_{km}^0 = \sum_n^N \sum_t^T S_{kt}^n C_{tm}^n$$

Solve for the modal response  $z_{mi}$  of mode  $m$  at each time step  $i$  or frequency step  $i$  using standard modal analysis techniques. The WFE Zernike polynomials  $Z$  at the focal plane can be obtained from following equation.

$$Z_{ki}^0 = \sum_m^M A_{km}^0 z_{mi}$$

Since the Zernikes are orthogonal over focal plane, the net WFE can be found from RSS of the polynomials where  $w$  is the conversion from polynomial magnitude to polynomial RMS.

$$WFE\_RMS = RSS(w_k Z_k^0)$$

The accuracy of this approach depends on the quality of the polynomial fit to optical surfaces in the mode shapes.. SigFit reports fitting errors from which the user can judge overall accuracy of the method.

### 2.3 Flow chart for System WFE in dynamics

Figure 3 shows the analysis flow to get system WFE response in dynamic analysis using a linear optics model

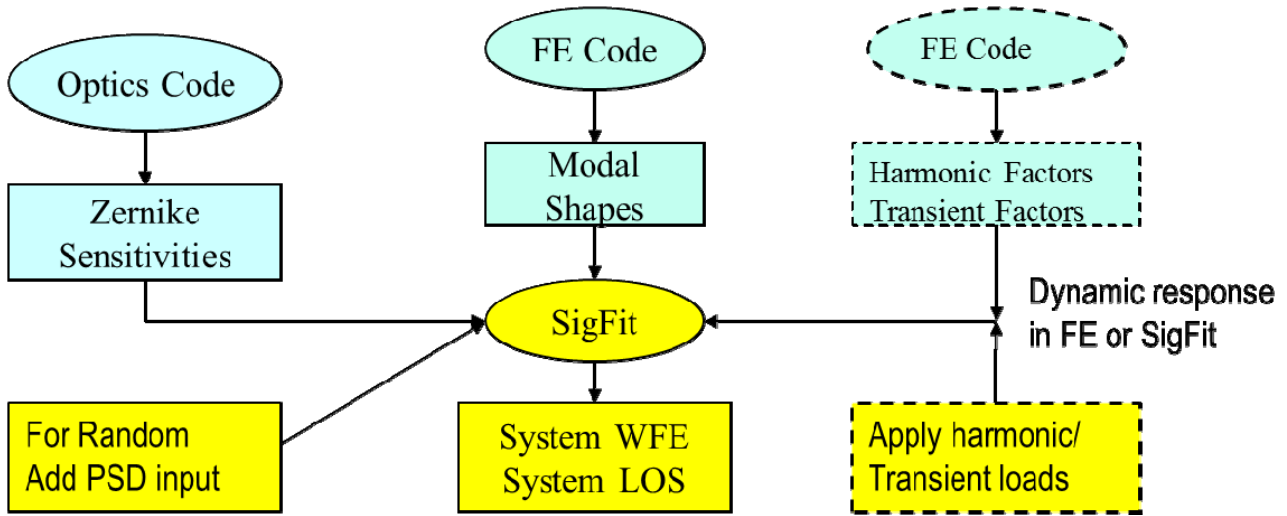
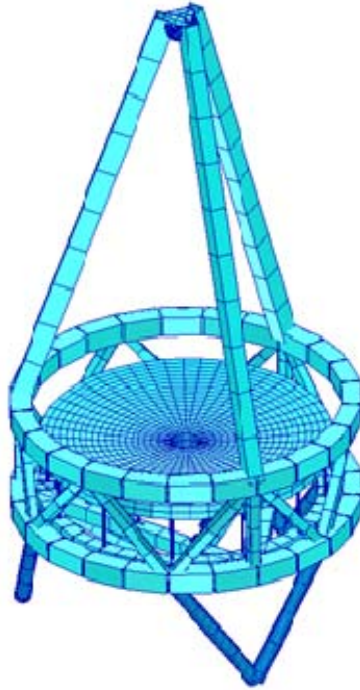


Figure 3: System WFE in dynamic analysis

### 3. EXAMPLE TELESCOPE

#### 3.1 Random base shake example

The simple telescope shown in Figure 4 is used to demonstrate the LOM technique using the flow chart in Figure 3.

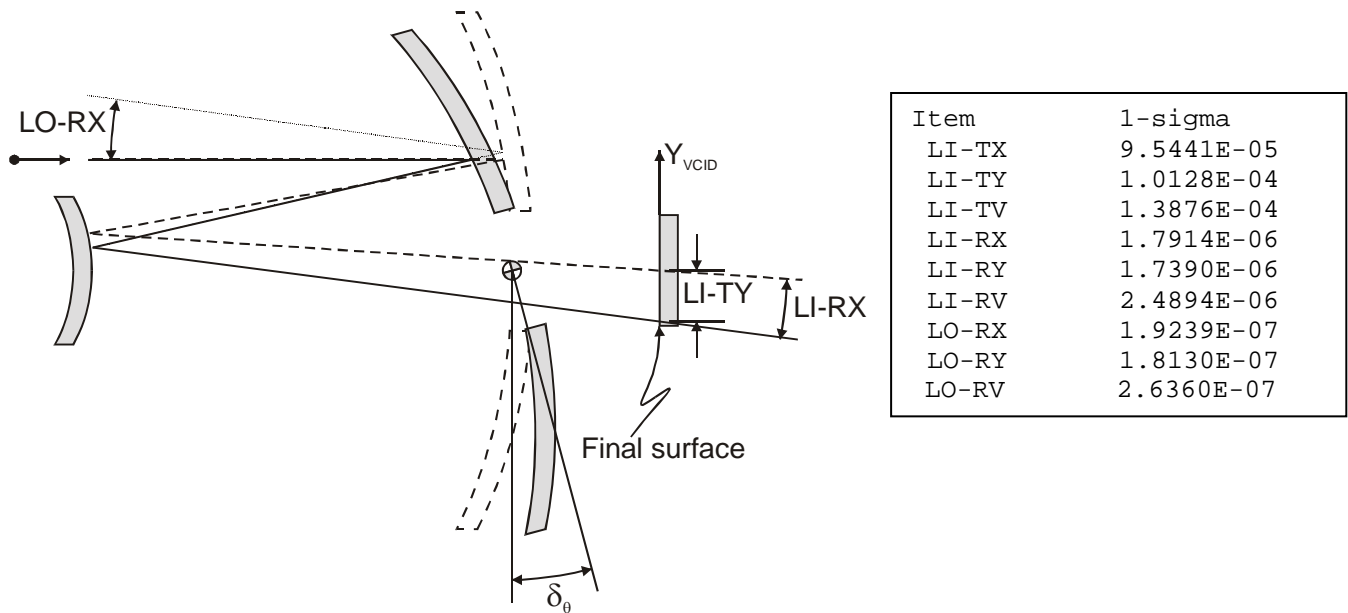


**Figure 4: Simple telescope FE model**

The mode shapes were calculated in Nastran and read by SigFit. The sine sweep harmonic base shake and the random response were calculated in SigFit. An alternative approach is to calculate the modal factors in the FE code for the sine sweep and pass those to SigFit.

#### 3.2 Line of sight (LOS) and modulation transfer function (MTF)

SigFit calculated the LOS coefficients via an internal ray trace for the chosen field point. The SigFit random response output included the 1-sigma response of the LOS both in object and image space as shown in Figures 5. A complete PSD response function output is also available.



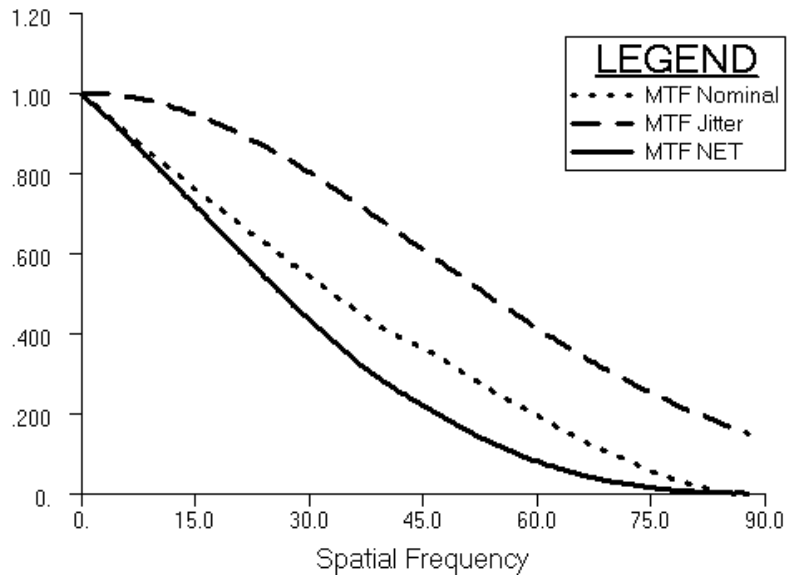
**Figure 5: Line-of-Sight 1-sigma response to base shake**

A very useful diagnostic tool is the SigFit output which gives each mode's contribution to random LOS as shown in Figure 6. Once the key modes are identified, the user can go back to the FE model, plot the strain energy density in the key modes to suggest design improvements.

Each modes % contribution to LoS JITTER PSD							
Mode	Freq	LI-TX	LI-TY	LI-TV	LI-RX	LI-RY	LI-RV
4	104.52	0.000	58.739	31.368	55.736	0.000	28.835
5	104.66	57.033	0.000	26.576	0.000	50.483	24.366
6	123.58	0.000	28.852	15.408	36.340	0.000	18.800
7	124.14	25.429	0.000	11.849	0.000	29.965	14.463
8	189.26	0.000	3.950	2.109	2.497	0.000	1.292
9	189.48	0.735	0.000	0.343	0.000	0.431	0.208
10	197.69	0.000	0.000	0.000	0.000	0.000	0.000
11	226.32	2.712	0.000	1.264	0.000	0.545	0.263
12	227.09	0.000	3.253	1.737	0.688	0.000	0.356
13	237.13	0.000	0.000	0.000	0.000	0.000	0.000

**Figure 6: Each modes contribution to LOS random jitter**

A nominal MTF curve obtained from the optical model was modified by SigFit to account for the image jitter effects from random response analysis<sup>[3]</sup> as shown in Figure 7.



**Figure 7: MTF curve reduced by effects of jitter**

### 3.3 System WFE in random analysis

The LOM approach was applied to get system WFE in random base shake. The SigFit results are shown in Figure 8.

Order	Aberration			1-sigma PolyMag (Waves)	1-sigma PolyRMS (Waves)
K	N	M			
			System WFE Total		0.021601
4	2	0	A Power (Defocus)	0.009881	0.005705
5	2	2	A Pri Astigmatism	0.011847	0.004837
6	2	2	B Pri Astigmatism	0.009238	0.003771
7	3	1	A Pri Coma	0.000866	0.000306
8	3	1	B Pri Coma	0.000810	0.000286
9	3	3	A Pri Trefoil	0.053645	0.018966
10	3	3	B Pri Trefoil	0.000087	0.000031
11	4	0	A Pri Spherical	0.005920	0.002647
12	4	2	A Sec Astigmatism	0.001636	0.000517
13	4	2	B Sec Astigmatism	0.001410	0.000446

**Figure 8: System WFE due to random base shake**

Figure 9 shows each modes contribution to WFE, the error polynomial fit to the mode, and the net error on the system WFE. This allows the user to judge the quality of the results. The modal contribution identifies the key modes just as in the LOS contribution.

Each modes contribution to System WFE PSD  
 Contrib = % of sys WFE from this mode  
 ModeErr = % fitting error in each mode = summed over all surfaces  
 = sum(ErrNet \* Surf\_Contribution\_to\_Mode\_WFE)  
 SysErr = Contrib \* ModErr = % Error in Sys WFE due to mode fitting

Lid	Mode	Freq	Contrib	ModeErr	SysErr
1	4	104.52	0.014	0.352	0.000
2	5	104.66	0.013	0.347	0.000
3	6	123.58	0.415	5.946	0.025
4	7	124.14	0.319	5.679	0.018
5	8	189.26	0.047	0.691	0.000
6	9	189.48	0.008	0.680	0.000
7	10	197.69	0.000	1.333	0.000
8	11	226.32	0.205	0.385	0.001
9	12	227.09	0.285	0.396	0.001
10	13	237.13	0.000	0.185	0.000
11	14	251.90	83.668	0.257	0.215
12	15	261.23	0.000	17.684	0.000
13	16	296.74	0.373	0.469	0.002
14	17	296.75	0.345	0.469	0.002
15	18	319.07	2.673	0.507	0.014

Figure 9: Each modes contribution to System WFE

### 3.4 System WFE in optimum design

SigFit can now write equations for system WFE in Nastran's bulk data format for optimization shown in Figure 10.

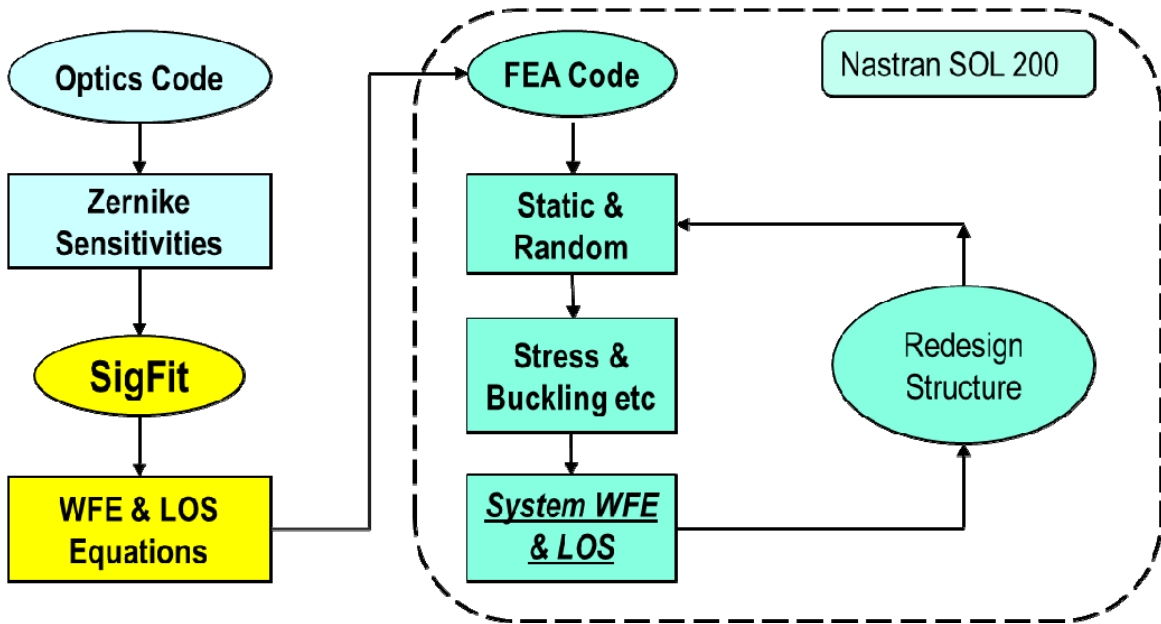


Figure 10: System WFE incorporated in optimization



With this feature, System WFE is a response that may be constrained or an objective to be minimized. SigFit writes a variety of optically important terms as bulk data equations <sup>[4]</sup>.

- System WFE polynomial terms written as MPC equation
- System net WFE RMS written as DRESP2
- System LOS written as MPC
- Surface Rigid-Body motion (with radial correction) written as MPC equation
- Surface displacements with RB subtracted written as DRESP1 & MPC
- Surface Zernike/other polynomial terms written as MPC equation
- Surface residual RMS written as DRESP2 equation

## 4. EXTENSIONS

### 4.1 Extensions of LOM approach

The linear optics model capability has been extended to all SigFit analysis types.

- 1) Fitting: system WFE polynomials and plots of WFE created
- 2) Active: corrected system WFE polynomials and plots of WFE created <sup>[5]</sup>
- 3) Harmonic Response: system WFE transfer functions are added to output (peaks noted)
- 4) Transient Response: system WFE time history added to output files (peaks noted)
- 5) Random Response: system WFE PSD functions, 1-sigma response, and modal contributors added to output
- 6) State Space equation output: system WFE include in response equations
- 7) Monte Carlo analysis: tolerance of system WFE to variations added to output <sup>[6]</sup>
- 8) Equation writing: system WFE equations written to finite element model (for optimization)

## 5. CONCLUSIONS

The automated linear optics model approach described above will allow design decisions to be based on the system response rather than individual component responses. In addition, the modal contributions provide the user with guidance on how to improve the system response.

## REFERENCES

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